

## LETTER TO THE EDITOR

# Stabilization in circularly polarized light: Floquet–adiabatic versus exact treatment

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**Abstract.** Calculations of the ionization yield obtained for the strong circularly polarized light ionization of  $m = 0, \pm 1$   $2p$  initial states from the so-called single Floquet state approximation are compared with the exact results reported by Gajda, Piraux and Rzązewski obtained via direct integration of the time-dependent Schrödinger equation. For  $m = -1, 0$  excellent agreement is found despite the fact that the pulse length is only 20 optical cycles. The differences between the two approaches for  $m = 1$  support the different mechanism of stabilization in that case.

Among various interesting phenomena occurring when atoms are illuminated by very strong laser fields (such as above-threshold ionization, high harmonic generation or multielectron ionization—for a review see [1–3]) the so-called stabilization, i.e. the decrease of the ionization yield with the increase of the light intensity for very strong fields [4–8] seems to be the most surprising. Two different mechanisms for the phenomenon have been proposed: the so-called adiabatic stabilization, which emphasizes the decrease with intensity of the widths of the states dressed by the field [5, 8], and the dynamic stabilization in which a coherent wavepacket stable against ionization is formed [9]. Obviously, the two mechanisms behave very differently when the pulse duration is changed: while the latter depends strongly on the pulse duration and should be effective only for short pulses, the former is by definition expected to be relevant only for slowly varying pulse shapes, i.e. for long pulses.

Most of the studies have been devoted to stabilization of the linearly polarized light, while up to very recently, only stabilization of the ground state of an atom in circularly polarized light (CPL) has been studied [5, 8, 10]. In a very interesting study, Gajda *et al* [11] (to be referred to as GPR) have discussed ionization from the  $2p$  hydrogen atom state showing that the phenomenon is strongly dependent on the azimuthal angular quantum number,  $m$ , of the initial state. They concluded that the strong stabilization observed for the  $m = 1$  state (in right-hand CPL) shows evidence for the dynamic stabilization mechanism. The results have been obtained by a direct integration of the time-dependent Schrödinger equation for a smooth pulse of duration of 20 optical cycles. It is interesting to see whether for such a short pulse one may still speak of the adiabatic stabilization mechanism and to test the applicability of the adiabatic treatment for such an extreme case. As far as we know there has been no such comparison presented before, neither for linear nor for circular polarization. This is the aim of this letter.

The hydrogen atom in the field of a circularly polarized radiation is described by (atomic units are used throughout this letter) the following Hamiltonian:

$$H = \frac{(\mathbf{p} + \mathbf{A})^2}{2} - \frac{1}{r} \quad (1)$$

with the vector potential,  $\mathbf{A}$ ,

$$\mathbf{A}(t) = \frac{F(t)}{\omega} (-e_x \sin(\omega t) + e_y \cos(\omega t)). \quad (2)$$

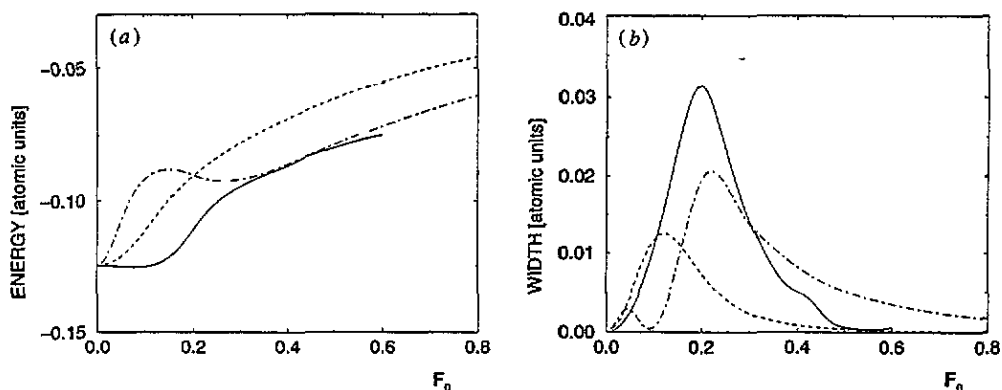
The standard Floquet approach finds the combined eigenstates of the (atom + laser light) Hamiltonian for different fixed values of the field amplitude,  $F$ . In our approach we use an equivalent procedure and diagonalize the time-independent Hamiltonian obtained by passing to the frame rotating with the frequency  $\omega$  of the CPL [12, 13] by means of the unitary transformation  $U = \exp(i\omega L_z t)$ :

$$H_R = \frac{p^2}{2} - \frac{1}{r} + \frac{F}{\omega} p_y - \omega L_z. \quad (3)$$

Note that customarily we drop the  $A^2$  term. The matrix elements of  $H_R$  are calculated in the Sturmian basis [8, 11], actually not using their configuration space representation, but rather algebraic properties resulting from the dynamical group symmetries of the Coulomb problem [14]. Due to the presence of the field (coupling to the continuum) the Hamiltonian, equation (3), supports no bound states, rather they are resonances. They are found in a standard way using the so-called complex rotation technique [15, 16]. In short, one rotates the coordinates and momenta according to  $\mathbf{r} \rightarrow \mathbf{r} \exp(i\theta)$ ,  $\mathbf{p} \rightarrow \mathbf{p} \exp(-i\theta)$ , where the rotation angle  $\theta$  can be chosen arbitrarily. The matrix of  $H_R$  becomes complex symmetric and explicitly dependent on the rotation angle  $\theta$ . As discussed, for example, in [15], the converged complex eigenvalues  $\epsilon_i = E_i - i\Gamma_i/2$  do not depend on  $\theta$ . Then  $E_i$ , the real part of  $\epsilon_i$ , is interpreted as a resonance position while  $\Gamma_i$  is the width of the resonance (inverse of its lifetime). Our approach has been tested extensively on the microwave ionization problem both for the two-dimensional [17] as well as for real three-dimensional atoms [18]; here we use the same procedures for a very different range of frequency and field amplitudes.

When the amplitude changes in time,  $F = F(t)$  the eigenenergies obtained at different  $F$  values correspond to instantaneous dressed states (corresponding to times when the amplitude is  $F$ ). Although the solutions of the time-dependent Schrödinger equation can be expanded on the Floquet basis for arbitrary pulse shape,  $F(t)$ , this expansion is very complicated in the general case and of no practical use.

The crucial point in the so-called single Floquet state approximation (SFS) is to assume that the atomic levels follow the changes of  $F(t)$ , adiabatically i.e. that only one Floquet state is populated at each time. The situation becomes complicated when the level in question undergoes avoided crossing, then identification of the state in the vicinity of such an avoided crossing is questionable. Two possibilities may happen. For a narrow (small energy gap) avoided crossing one may assume a diabatic passage when the change of  $F(t)$  is much faster than the time-scale,  $\tau$  set by the inverse size of the avoided crossing gap. For large avoided crossing, on the other hand, an adiabatic passage is possible provided the inverse condition is met, i.e.  $\tau \ll \frac{1}{F} \frac{dF}{dt}$ . Then it is sufficient to follow the single dressed (Floquet) state which is a smooth continuation of the initial state. When avoided crossings of intermediate size appear, the crossing is neither diabatic nor adiabatic, leading to the population of several states. The identification of the important state becomes questionable and the SFS breaks down.



**Figure 1.** Floquet energies ((a) real parts) and the corresponding widths ((b) twice the imaginary part taken with negative sign) as a function of the top amplitude of the electric field of the  $\sigma^+$  polarized light of frequency  $\omega = 0.25$  for the Floquet states adiabatically continued from the  $2p$   $m = -1, 0, 1$  states represented by full, broken and chain curves, respectively.

SFSA is a common assumption used in Floquet-type calculations [8, 19]. The energy of the state and its width change with  $F(t)$  providing information about the non-resonant decay of the atomic population to the continuum. Then the probability of ionization after the microwave pulse of duration  $T$  is given simply by

$$P(T) = 1 - \exp \left\{ - \int_0^T \Gamma[F(t)] dt \right\} \quad (4)$$

where  $\Gamma$  is the width of the important single Floquet state.

Note that the method requires finding a *single* eigenvalue for a set of  $F$  values up to some  $F_{max}$ . Once these are found one may calculate  $P(T)$  by simple integration for *arbitrary* pulse shape and its duration. We use the Lanczos algorithm which allows us to very effectively find the few eigenvalues in a desired energy interval, locating the desired eigenvalue by calculating the overlap between the eigenfunctions obtained in two consecutive diagonalizations.

We now present the results of our calculations, done in the conditions used in GPR, namely the ionization of the  $2p$  state of the hydrogen atom (in any of its three Zeeman sublevels) by a circularly polarized field of frequency  $\omega = 0.25$ . At such high frequency, only one photon is needed to ionize the atom, although multiphoton processes play a key role in the stabilization mechanism.

Figure 1 presents the Floquet real energies (figure 1(a)) showing the effect of AC Stark shift and the width of the resonances (figure 1(b)). Note that for the field amplitudes corresponding to the strong, stabilization region (above  $F = 0.3$ , see below) the real parts of resonance energies for  $m = 1$  and  $-1$  follow each other closely. This is a manifestation of the 'high-frequency, high-intensity' limit. As discussed in [22] energies of states corresponding to some absolute value of  $m$  coincide in the time-averaged Kramers-Henneberger potential. In contrast, the widths of the states, as exemplified in figure 1(b) behave quite differently since they originate from the residual coupling breaking the 'high-frequency, high-intensity' symmetry.

Notice also that the curves corresponding to  $m = -1$  in both figures 1(a) and (b) are plotted only up to  $F = 0.6$ . For higher  $F$  values, the state in question undergoes a series of avoided crossings making the physical state identification quite ambiguous. The signature

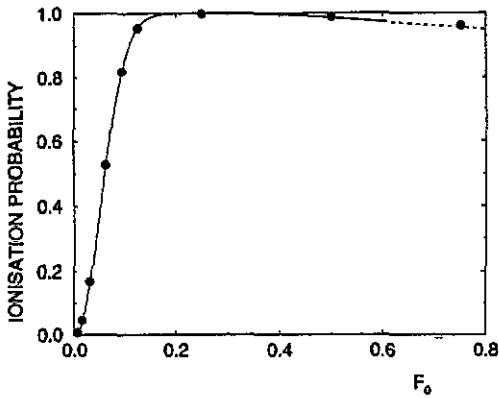


Figure 2. Ionization probability from the 2p state with  $m = -1$  as a function of the maximal electric field for the sine-square  $\sigma^+$  polarized pulse of duration 20 optical cycles and the frequency  $\omega = 0.25$  au. Full circles: exact data from GPR, full curve: present results of the single Floquet state approximation (SFSa) extended to higher fields (broken curve) by a strictly adiabatic procedure—see the text.

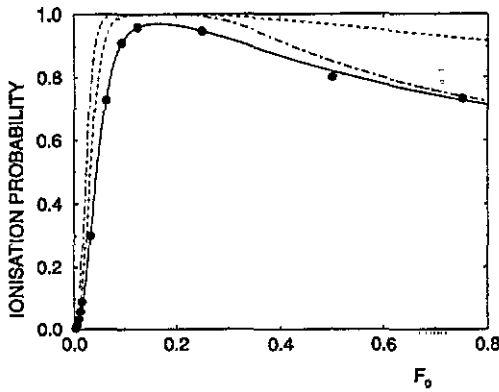


Figure 3. Same as figure 2 but for  $m = 0$ . The broken curve corresponds to sine-square pulse of duration 40 optical cycles while the chain curve presents the results obtained for a 'flat-top' pulse with the same rise and turn-off time as the 20 cycles sine-square pulse.

of the first of these crossings is seen in figure 1(a) as a deviation from the  $m = 1$  energy line (see the discussion in the preceding paragraph).

The data presented in figure 1(b) are then used to calculate the ionization yield. This allows us to compare the exact ionization yields obtained by GPR for the pulse of the form

$$F(t) = F_0 \sin^2(\pi t/T) \quad (5)$$

(where  $T$ , the pulse duration is equal to 20 optical cycles) with the results obtained by applying (4). Figures 2 and 3 show this comparison for  $m = -1$  and  $m = 0$  Zeeman sublevels of the 2p state. Bearing in mind that the pulse used by GPR in their calculations is extremely short (only 20 optical cycles), such a good agreement between exact and adiabatic SFSa is quite surprising. The broken curve in figure 2 represents the yield obtained for maximal amplitudes in the regime of strong avoided crossings (i.e. when identification of the single important state is dubious) assuming adiabatic following of levels. We have checked that the corresponding yield is not sensitive to a different selection of the state followed, a signature of the fact that the ionization yield is determined during the pulse rise for  $F$  values smaller than 0.6.

The agreement between SFSa and exact results is the best numerical proof that both a weak (for  $m = -1$ ) and quite strong (for  $m = 0$ ) stabilization observed is purely adiabatic in origin. This is further exemplified by broken and chain curves in figure 3. The broken curve corresponds to the pulse of the same shape but twice the duration. Practically total ionization occurs, since the pulse rises slow enough (in this range of maximal amplitudes  $F_0$ ) for the ionization to be completed during the turn on of the field amplitude [20]. The

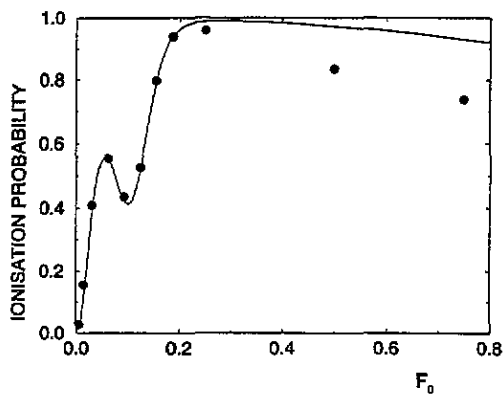


Figure 4. Same as figure 2 but for  $m = 1$ . Note the disagreement between the single Floquet state approximation (SFSA) and exact results for strong fields. For further explanation see the text.

chain curve represents a pulse of twice the length, but of 'flat-top' shape, in which two parts of the sine-square pulse are joined by the constant amplitude part of 20 optical cycles. For small maximal amplitudes this pulse ionizes the atom quite effectively, for large  $F_0$  in the stabilization region it gives only slightly larger ionization than the sine-square pulse of half its duration. Since both pulses have the same rising and turn-off phases the result exemplifies the very slow ionization rate corresponding to the high-field flat part of the flat-top pulse.

Figure 4 presents a similar comparison for  $m = 1$ . For sufficiently low fields (up to the maximum ionization yield) again an impressive agreement between exact results of GPR and the SFSA is observed. Note, in particular, how well the stability window (local minimum in figure 4) is reproduced. For higher maximal amplitudes a discrepancy occurs. While SFSA predicts weak adiabatic stabilization (in fact, very similar to that observed for  $m = -1$ ) exact results show a much stronger effect. The latter result has been explained by GPR as a manifestation of the dynamic stabilization of Fedorov and Movsesian [9], i.e. a creation by the strong field of the wavepacket of populated highly excited states due to a phenomenon quite similar to population trapping [21]. Clearly, since other than the initial (adiabatically followed into the strong-field regime) state become significantly populated in this case, the SFSA must fail. Still the SFSA results give us interesting information—the comparison in figure 4 shows the relative importance of purely adiabatic mechanism (SFSA) and of dynamical stabilization—the exact results contain both the effects combined.

The interpretation presented can be further supported by an analysis of Floquet states shifts for  $m = 1$  and different  $n$  values (not shown). Around  $F = 0.1$  (local minimum of the ionization curve in figure 4) the  $n = 2$  level undergoes a quite broad avoided crossing with the  $n = 3$  state (also states with a higher  $n$  values with the same  $l$  and  $m$  are involved in the same multilevel broad avoided crossing). Since this avoided crossing is broad we pass it adiabatically. This is clearly justified for not too large  $F$  values, when  $F = 0.1$  is close to the top value,  $F_0$ , of the pulse, i.e. when changes of the field amplitude are relatively slow. For larger and larger  $F_0$ , the region around  $F = 0.1$  is passed faster and faster during the pulse rise and the adiabaticity is broken creating the difference between the exact results and SFSA. In the real world, non-adiabatic transitions occurring around  $F = 0.1$  build a wavepacket of different  $n$  states yielding the dynamical stabilization in complete agreement with the interpretation already given in GPR.

To conclude, we have demonstrated, by comparison with the recent exact results of GPR, the usefulness of the Floquet adiabatic picture even for extremely short laser pulses. A comparison with exact results enables deeper understanding of the dynamics and allow

to distinguish between two competing (or rather collaborating) mechanisms of atomic stabilization.

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