nizing BMP signals which normally act as negative regulators of *chordino* expression.

In addition, *chordino* mutants display a dorsally expanded *bmp-4* expression domain[3], indicating that during normal development *chordino* has a negative influence on *bmp-4* transcription in dorsolateral regions. Again, this negative regulation might be mediated by BMPs themselves, which normally act as positive regulators of their own expression[3].

It has been shown that *Xenopus* Chordin inhibits the antagonizing Bmp-4 activity by direct binding to the Bmp-4 protein, thereby preventing receptor activation[4]. Our data demonstrate that this inhibition at the protein level is potentiated by two synergistic feedback loops which lead to a derepression of *chordino* transcription and an inactivation of *bmp-4* expression.

Our findings demonstrate that *chordino* is required during early dorsoventral patterning of the zebrafish embryo, where it functions as an antagonist of ventralizing BMP signals. This supports the notion that mechanisms of early dorsoventral patterning are conserved between vertebrates and invertebrates[13].

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Have quantum scars been observed?

Wilkinson *et al.*[1] have reported observations of oscillations in the current of a resonant tunnelling diode in a magnetic field. They proposed that this provides the first direct experimental evidence for quantum 'scarring' in a real chaotic system. But we believe that Wilkinson *et al.* have not observed quantum scars in the usual sense. They considered a far-from semiclassical regime where current theories of scarring are not valid. Demonstrating these effects in a solid-state device is a tremendous achievement. However, we believe that even in the semiclassical limit this type of experiment does not go beyond other experiments on atoms in magnetic fields as a probe of scarring.

A quantum particle travelling from A to B experiences all possible paths in between. The wave-like nature of a quantum particle means that each path has a phase and so neighbouring paths interfere with each other. The combined contributions of these paths build up the wavefunction, which provides a complete description of the quantum dynamics of the particle. At high energies, in the semiclassical limit ($\hbar \to 0$), phases associated with different paths are large. Hence, neighbouring paths can rapidly fall out of phase and their contributions cancel by destructive interference. When the typical motion is a chaotic trajectory, this should produce wavefunctions with no structure other than a random grainy pattern.

But Heller[2] found that for $\hbar \to 0$, quantum states of a chaotic system show 'scars', concentrations near the paths of unstable, isolated periodic orbits (classical orbits which retrace themselves and are isolated if there is no other periodic orbit of similar phase nearby). The issue of how and when quantum states are scarred in the semiclassical limit has sparked a lively debate. The mathematical theories all assume that the periodic orbits are isolated, though more than one isolated orbit can scar a single state.

Wilkinson *et al.* have probed the 25 or so lowest quantum levels above the ground state. By lowering the energy, the phases of the classical orbits are made small, so that they remain in-phase. Hence, the neighbourhoods of topologically distinct periodic orbits become so large that they overlap and so cease to be isolated.

In the experiment, the electronic motion is confined by a magnetic field to a cylindrical energy surface whose ends are the two barriers of the well. Wilkinson *et al.* found that the tunnelling is dominated by four single eigenstates, roughly 'linear' in shape with $N \approx 10$ oscillations along the magnetic field. However, the energy surface supported only $n \approx 2$–3 oscillations perpendicular to the field, so each 'linear' eigenstate is supported by a large fraction of the Poincaré surface on the emitter wall, including a cluster of short periodic orbits of similar action that traverse the length of the cylinder.

This similarity of action is analogous to back and forth motion close to the axis of a long, thin cylinder. Most of the action comes from the longitudinal motion, with smaller differences due to the transverse components. The periodic orbits become isolated only when we can resolve the differences in action (typically for $n > 10$). For small n (non-isolated case), sequences of individual 'linear' states, roughly aligned with the magnetic field, are ubiquitous[1,3]. These states cannot distinguish between the different periodic orbits. The stability parameters of the classical dynamics are essentially irrelevant, in contrast to an isolated scar. They are essentially a quantum phenomenon, the so-called adiabatic separability[4], more relevant in the deep quantum regime than an interpretation in terms of scars. A linear state ($n \approx 1$) is shown in Fig. 1a, and Fig. 1b shows a scar in the semiclassical regime ($n \approx 30$).

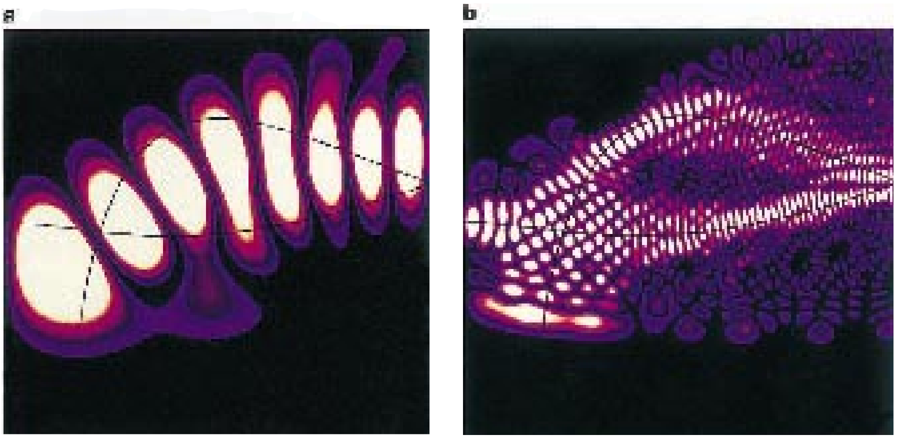

Figure 1 Quantum eigenstates which dominate tunnelling for: **a,** $n \approx 1$ far from the semiclassical limit showing 'linear' quantization; and **b,** $n \approx 30$ showing scarring by the S_1 unstable periodic orbit observed in wide-well experiments. We ensured that both states corresponded to exactly the same set of classical periodic orbits, differing only in effective size of $\hbar$ by using a scaling property of the dynamics.

Periodic-orbit effects have been extensively investigated using highly excited atoms in magnetic fields. The classic 1969 experiment[5] revealed a set of evenly spaced modulations in atomic photoabsorbtion spectra near the ionization threshold, associated with quantum states scarred by an unstable periodic orbit.

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Fromhold et al. *reply* — Unstable but periodic classical orbits are of fundamental importance in quantum chaology[6,7]. They produce regular clustering of the energy levels[6] and certain states exhibit regions of enhanced probability density or ‘scars’ in the neighbourhood of the classical periodic trajectories[2]. The scarring effect seems to have been unexpected in view of the instability of the periodic orbits[8]. The typical trajectory is chaotic and irregular, which would be expected to correspond to wavefunctions exhibiting irregular and diffuse patterns of probability density. We recently showed[1] that scarred and unscarred states can make very different contributions to a physically measurable quantity, in our case the tunnel current through a semiconductor device.

There is much published work on the photoabsorption spectrum of highly excited hydrogenic atoms in a strong magnetic field, which show modulations due to the effects of unstable periodic orbits in the classically chaotic regime. Owing to the complexity of the spectra at high excitation energies in the semiclassical limit, this has provided only indirect evidence for the scars confined to individual eigenstates[9–11]. The 1969 experiment[5] referred to by Monteiro *et al.* gave a beautiful demonstration of the role of unstable periodic orbits in quantum chaology, but had insufficient resolution to distinguish between the effects of wavefunction scarring and energy-level clustering on the photoabsorption spectra. However, Müller and Wintgen[12] noted that the scarring phenomenon survives the semiclassical limit and found that, at low excitation energies, wavefunctions were scarred by short-period orbits. It is this regime that our resonant tunnelling experiments were designed to investigate[1].

Our theoretical studies of the electronic levels in a 120-nm-wide quantum well of a resonant tunnelling diode in the chaotic regime revealed subsets of states, strongly scarred by particular periodic orbits[13], and also indicated that the current through the device was dominated by tunnelling via the scarred states in the well. But our experiments on the 120-nm-wide quantum well[14] could not resolve discrete levels because of broadening by scattering processes.

Our work on a 22-nm-wide quantum well[1] with clearly resolved energy levels showed that peaks in the current–voltage characteristic of the device corresponded to electrons tunnelling into the scarred states of the quantum well. This was related directly to the concentration of the probability density of these states close to a cluster of three very similar unstable scarring orbits with almost identical periods. By contrast, the unscarred wavefunctions of adjacent levels have irregular antinode patterns, which inhibit the tunnel process (Fig. 2). Further, the energies of the scarred states and their relation to orbital period were given accurately by the semiclassical quantization rule[4,15]. The results are from exact numerical calculations, valid for all energies, and are independent of approximate general theories of wavefunction scarring, which break down in the quantum limit. Note also that the quantum well exhibits particularly strong scarring effects because of its special dynamical properties, such as the absence of spatially closed but aperiodic orbits, which are not shared by atoms.

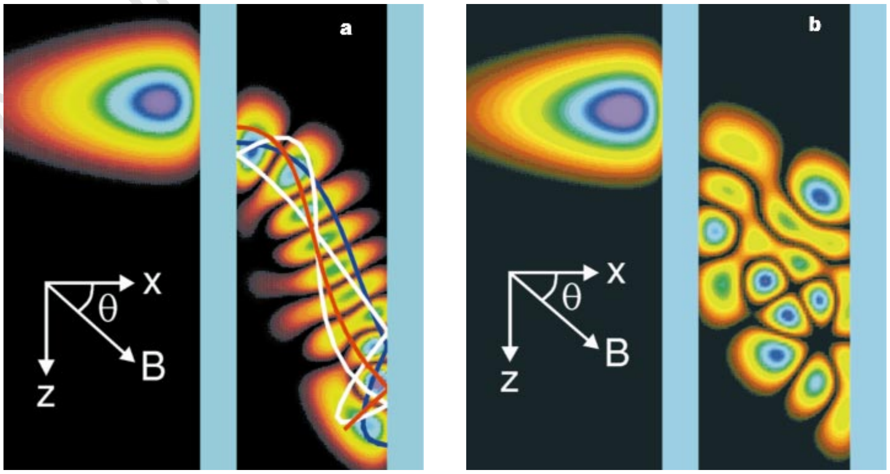

Figure 2 Probability density plots for two neighbouring eigenfunctions of the 22-nm-wide quantum well described in ref. 1. Probability is represented by a linear colour scale with $P=0$ in black, increasing linearly from ‘warm’ to ‘cold’ colours. The probability density of a localized state in the emitter accumulation layer is also shown. Blue rectangles indicate tunnel barriers. The plots are drawn in the x–z plane, where the x-axis is perpendicular to the confining barriers. A magnetic field of 37 T is applied in the x–z plane at an angle $\theta=40°$ to the x-axis. **a,** One of the scarred states identified in ref. 1 with energy $E_n=309.25$ meV. The three scarring orbits are shown overlaid. **b,** The probability distribution for the next highest energy level with $E_{n+1}=315.5$ meV reveals no trace of scarring. A bias voltage of 668 mV was applied across the resonant tunnelling diode containing the well and the effects of conduction-band non-parabolicity were included in the calculation[1].

Monteiro *et al.* argue that our results are not attributable to “scars in the usual sense”, as they do not distinguish between the different short-period orbits of the cluster. But this view is not supported by the literature or generally accepted. Semiclassical scarring of subsets of states by clusters of similar orbits has been shown[4]. For non-isolated orbits, Heller[2] refers to super-scars resulting from the overlap of many scars and comments that “increasing the density of similar periodic orbits can only enhance the scarring effect”.

The scarring effect has also been studied for the classical electromagnetic eigenmodes of a microwave cavity[16]. Here the scarring orbits are the periodic trajectories of rays reflecting around inside the cavity. In this case it was observed that the association of wavefunctions with periodic ray orbits emerges even at low frequencies when the electromagnetic wavelength is only roughly one-quarter of the cavity dimension. Analogous conditions hold in our resonant tunelling diode where the antinode quantum numbers, (ν) for the observed scarred states ($\nu\approx6$–9) are relatively small, as the de Broglie wavelength of the electrons is roughy one-quarter of the width of the quantum well.

But this in no way invalidates the physical concept of scarring as an imprint of an unstable periodic orbit on the quantum state of a classically chaotic system. The doubts raised by Monteiro *et al.* originate from their rather restricted view of scarring and are not substantiated in the literature.

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