Comment on "New States of Hydrogen in a Circularly Polarized Electromagnetic Field"

In a recent Letter, Kalinski and Eberly [1] discuss states of the hydrogen atom dressed by a circularly polarized microwave field, localized on an unstable equilibrium point, stating that their localization is ''purely quantum mechanical in character." This statement is incorrect, at least for the model discussed and for the numerical examples shown in the figures. The authors use the standard pendulum approximation [2] for the description of the resonance between the microwave field and the electronic motion. There are two fixed points then (one stable and one unstable) in the rotating frame—corresponding to circular periodic orbits in the laboratory frame—and the whole dynamics is integrable. In such a system, localization around the unstable point is of purely *classical* origin: If the total energy is close to the energy E_u of the unstable equilibrium position, the motion slows down near the unstable point. The time-averaged probability density of the pendulum is strongly peaked near it, simply because it spends more time there. In quantum mechanics, the eigenstates of the pendulum essentially have the same localization properties (as can be seen from the semiclassical WKB approximation): The states with energy close to E_u are strongly localized around the unstable fixed point—even in the limit $\hbar \to 0$—since this localization is classical in character.

The authors of [1] present the condition for the existence of ''purely quantum-mechanical'' stabilized states, Eq. (5) of [1], namely that the scaled microwave amplitude $\mathcal{E}_{sc}$ should satisfy $\mathcal{E}_{sc} < \mathcal{E}_{cr} = 3/4m_0$ for sufficiently large m_0 (m_0 is the projection of the angular momentum on the z axis). As an example, they present in Fig. 2(b) a localized state at very low $\mathcal{E}_{sc} = 0.0064$ (where the localization is classical in character, see above) for $m_0 = 18$.

The quantum scar effect refers to an increased probability density around an unstable periodic orbit (or here an unstable fixed point) embedded in a large chaotic sea [3]. While the classical motion is delocalized, some quantum states are scarred by the unstable periodic orbit because of quantum interferences. This quantum phenomenon disappears in the semiclassical limit [3]. Hence the description of the classical localization observed for the pendulum system or the hydrogen atom in low amplitude circularly polarized microwave field (where the classical dynamics is essentially regular) as in Fig. 2(b) of Ref. [1] using the notion of scars is misleading.

When the microwave field is significantly increased, a substantial part of the classical dynamics turns chaotic, especially around the unstable fixed point, and scarring can be observed. However, the quoted condition (5) is incorrect. Our tests, performed by an "exact" diagonalization of the Hamiltonian using a Sturmian basis expansion and complex coordinate rotation (for details, see [4]) indicate that ''scarred'' localized wave packet eigenstates persist up to much higher microwave amplitudes. Examples are presented in Fig. 1; the corresponding microwave amplitudes are several times bigger than $\mathcal{E}_{cr}$ for the case $m_0 = 60$, far in the semiclassical limit. Similar plots are commonly observed for a wide variety of m_0 and $\mathcal{E}$ values, far above the condition of [1].

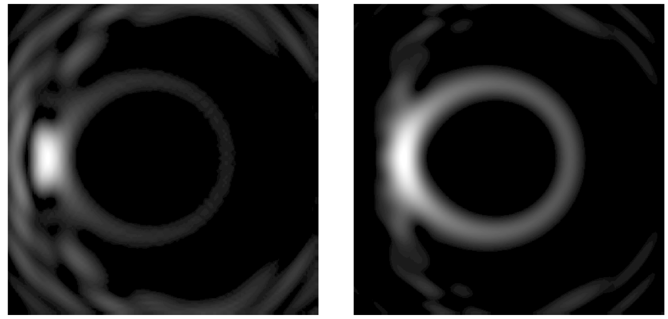

FIG. 1. Eigenstate localized on the unstable fixed point at scaled amplitudes $\mathcal{E}_{sc} = 0.057 = 4.6\mathcal{E}_{cr}$ (left) and $\mathcal{E}_{sc} = 0.07 = 5.6\mathcal{E}_{cr}$ (right). The scale is ± 5000 Bohr radii in both directions.

Let us note finally that [1] gives a wrong condition for the stable point turning unstable which should read $\mathcal{E}_{\text{stable}} = (1/3)^{4/3}/2 = 0.115\,56$ instead of 0.1068, and that in the expressions of $\mathcal{E}_0$ p. 2422, one should read $\omega^{4/3}$ instead of $\omega^{-4/3}$.

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