quantum limits in optical imaging

Projet européen : "Quantum Imaging"

IMAGES

are an important subject in optics,
and in information technologies

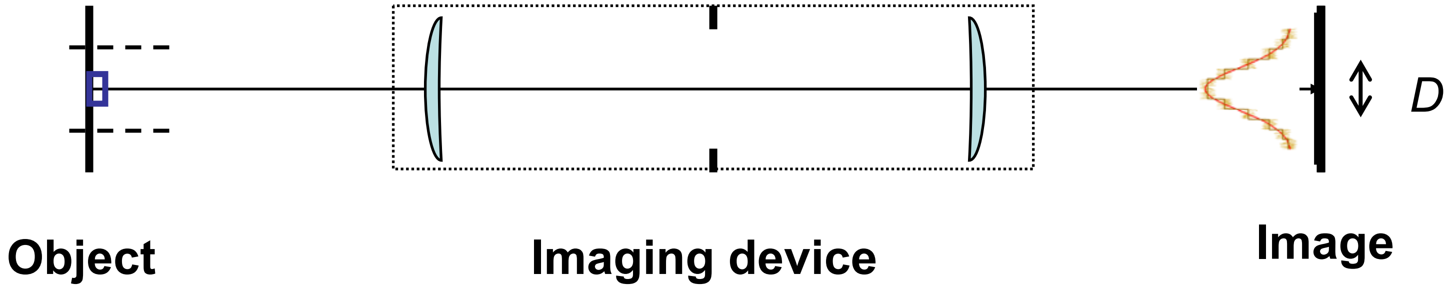
- *multiplexed information, monitored by multi-pixel detectors*
- *because of intrinsic parallelism of information*

Because the quantum nature of light introduces
quantum fluctuations,
which affect the sensitivity and/or resolution
of measurements
one can raise the following questions :

is there a Quantum Limit to resolution in optical images ?

is there a Quantum Limit to information extraction from optical images ?

1 : Resolution in optical images



XIX° century physics *sensors (eye, photographic plate) are not really linear*

the resolution is limited by the spot size *D : Rayleigh criterion*

Diffraction theory gives the answer for the ultimate limit : the wavelength

XX°, XXI° century physics sensors (diode arrays, CCD cameras) ***are very linear and have a wide dynamic range.***

If one is able to measure perfectly the image
one can determine exactly the object by deconvolution (inverse problem)

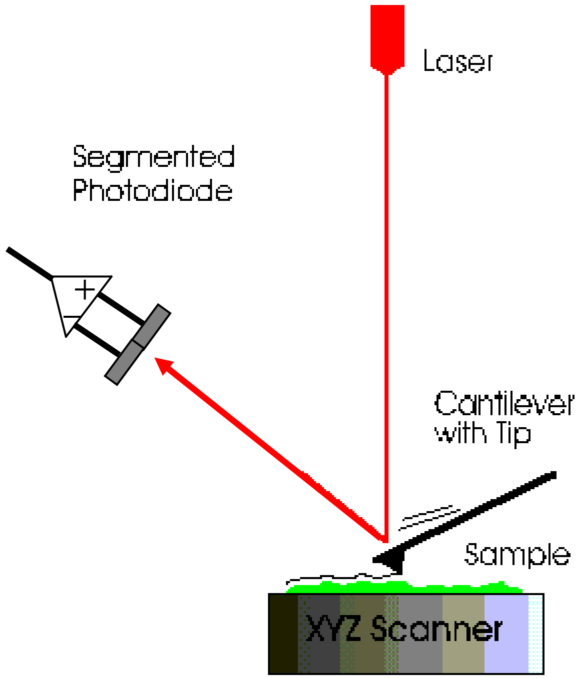
resolution is limited by noise in the read-out of the image

It is the Quantum noise on the image
which ultimately limits
the accuracy of the object reconstruction

2 : Information extraction from optical images

There are many examples of optical measurements where one does not want to know all the details of the field distribution in the transverse plane,
only the modification of the image
induced by parameters that one wants to determine

Example 1 : beam positioning



Atomic Force Microscope

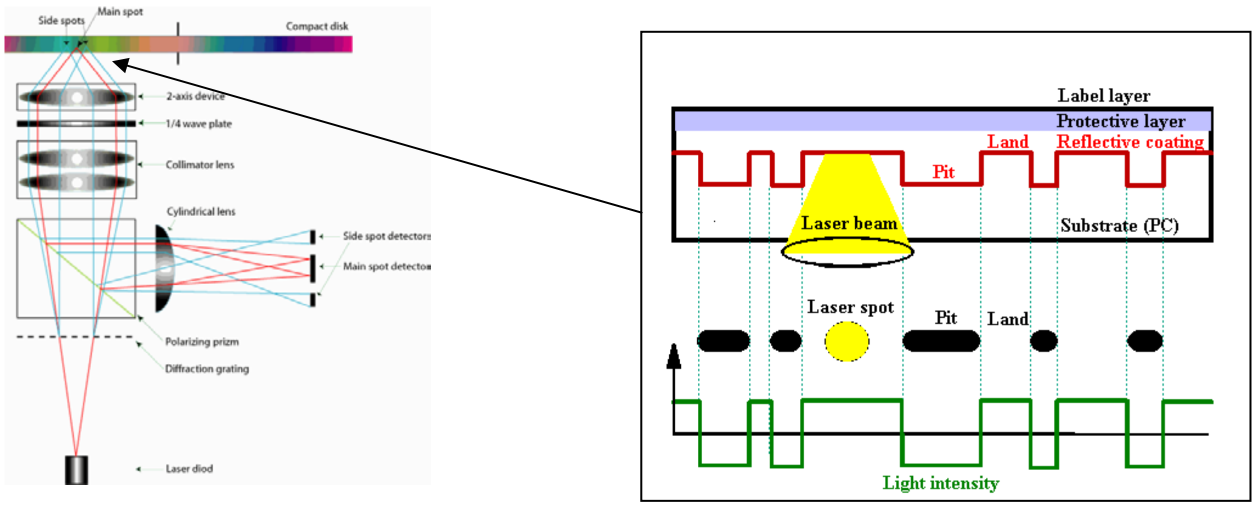
the AFM cantilever position change is monitored through the deflection of the laser beam

In such a case, the beam shape is unchanged, only its position is not known

10 nm beam displacement currently measured

what is the lower limit in such a measurement ?

Example 2 : digital optical data read-out



one only wants to detect the presence or absence
of pits of known depth and width
in the disc surface

so far, the density of bits is limited to 1 bit/λ^2

can one do better ?

Quantum aspects in optics

in linear optics,
quantum mean values obey the laws of classical electrodynamics

⇒ *quantum effects are effective in :*

- **fluctuations**
  - *spatial squeezing (local noise reduction) improvement of sensitivity and resolution in images*
  - *noiseless amplification of an image*
- **correlations**
  - *local spatial quantum correlations*
  - *spatial EPR entanglement*
- **nonlinear effects**
  - *two photon absoprtion with quantum light may lead to a better resolution in lithography : "quantum lithography"*

There are two levels of quantum effects :

- the "standard" quantum effects

In standard laser beams
the corpuscular character of light introduces noise
in photocurrent : the shot noise

⇒ limit usually called "standard quantum limit"

- beyond the "standard" quantum effects

Heisenberg inequalities limit the product of variances of quantum fluctuations
not a single variance

⇒ there exists "non classical states of light" allowing us to go below the standard limit on any single measurement

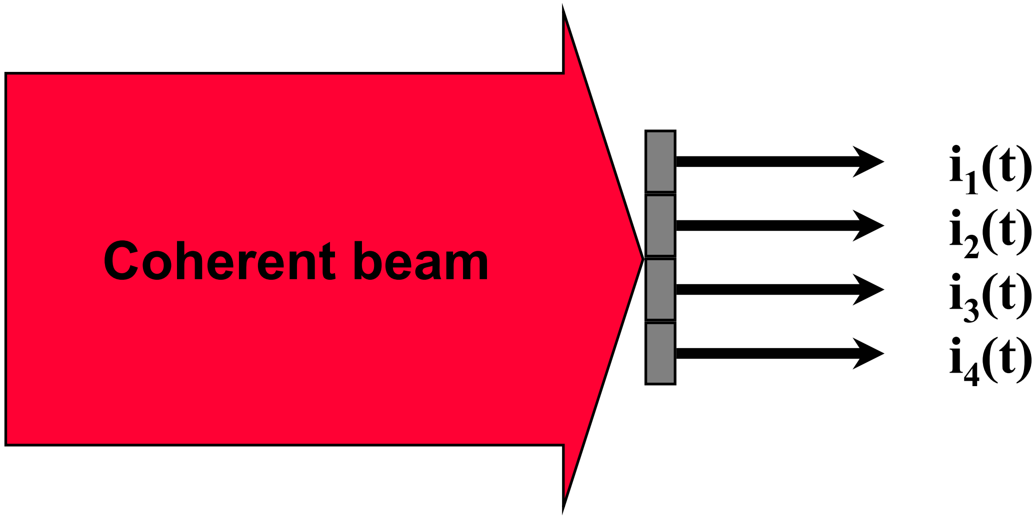
squeezed states, sub-Poissonian states, intensity correlated states ...

-I-

Standard quantum limit in images

(using ordinary laser beams)

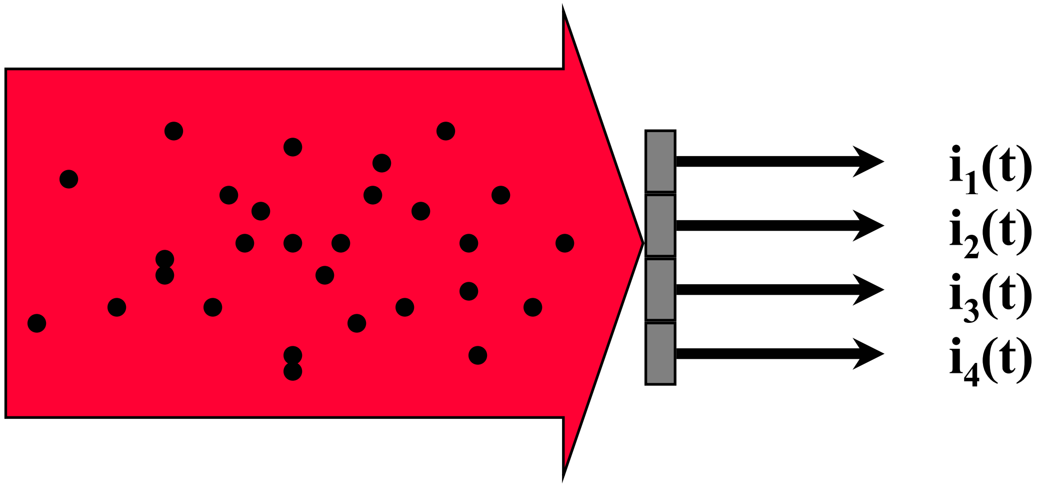
What are the quantum fluctuations and correlations measured on pixels when one uses a **coherent beam of light (standard laser beam) ?**



- there exists a «local» shot noise on each pixel

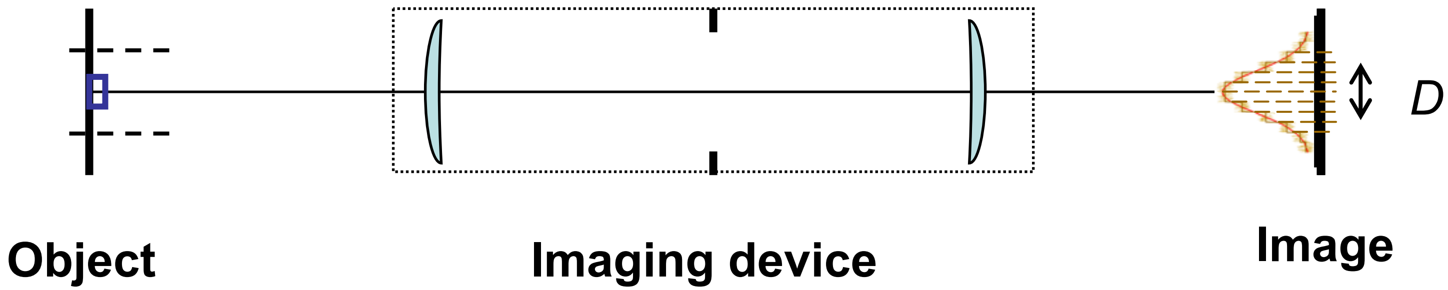
- there are no spatial correlations between different pixels

⇒ these features can be "explained" by considering that a coherent beam of light is «composed» of photons **randomly distributed in space and time**



A.

Standard Quantum Limit in optical resolution

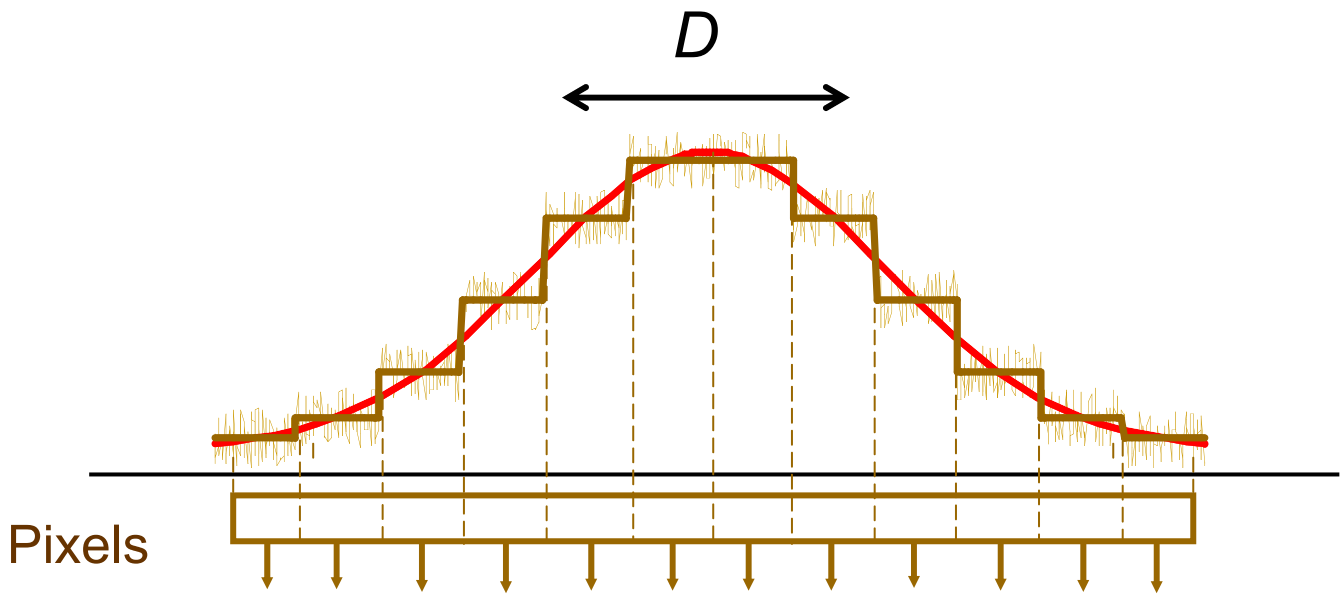


There are two steps in imaging :

(1) ⇒ *The recording of the image*

(2) ⇒ *The object reconstruction from this information*

(1) Standard Quantum Limit in image recording



If one decreases the pixel size, the sensitivity to details increases until one reaches the limit where one cannot distinguish the information given by two nearby pixels, because of the presence of shot noise

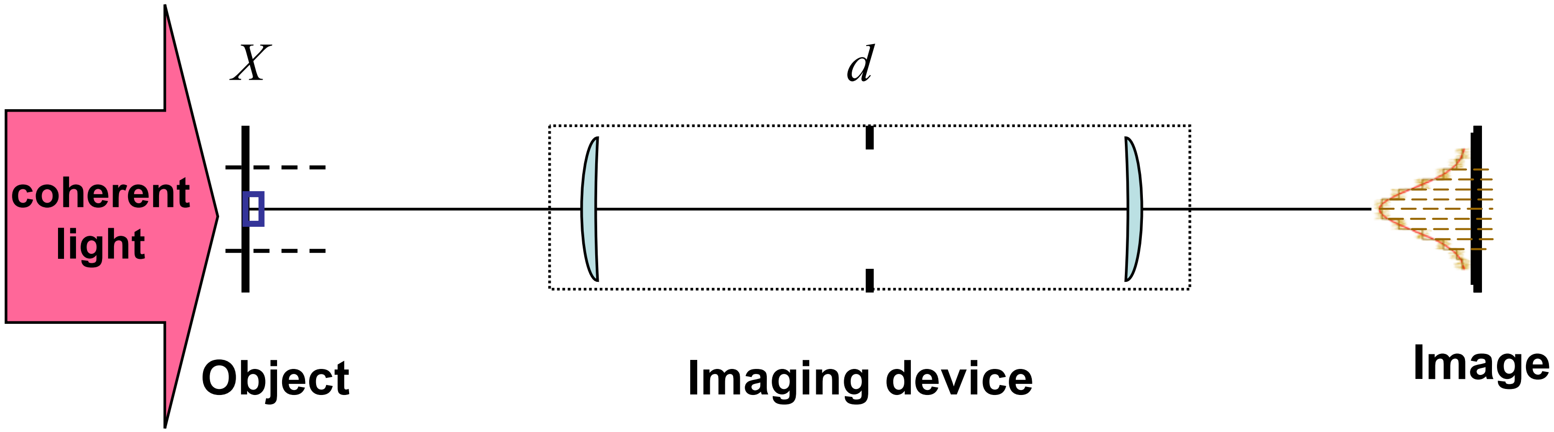
The minimum pixel area A_{pixel} allowing to monitor a detectable detail can be shown to be :

$$A_{pixel}^{\min} \approx \frac{D}{\sqrt{N\ (photons\ /m^2)}}$$

A detector with pixels having a finite size will be able to extract the total information available from the image

(2) Standard Quantum Limit in object reconstruction from image

Bertero, Pike Opt. Acta **29,** 727 (82)



Because of linearity, there exists eigenstates f_n of the imaging device, with transmission t_n

$$E_{object} = f_n \quad \Longrightarrow \quad E_{image} = t_n f_n$$

$$E_{image} = \sum_n c_n f_n \quad \Longrightarrow \quad E_{object} = \sum_n \frac{c_n}{t_n} f_n$$

if the coefficients c_n are perfectly known,
Object shape can be reconstructed **without limitation due to diffraction**

Local Shot noise $\Longrightarrow$ the knowledge of coefficients C_k is not perfect

One can show that : $(\Delta c_n)^2 = \text{constant}$

n small : $t_n \approx 1$

n large : $t_n << 1$

$$E_{image} = \sum_n c_n f_n \quad \Longrightarrow \quad E_{object} = \sum_n \frac{c_n}{t_n} f_n$$

the uncertainty δc_n is amplified in the object reconstruction for small values of t_n

shot noise prevents to know the coefficients of the decomposition for

$$n > n_{max}$$

$\Rightarrow$

Standard quantum limit in image reconstruction

$$\delta x_{\min} \approx X / n_{\max}$$

n_{max} depends on Shannon (or Fresnel) number $dX/\lambda f$ of the set-up

one can reconstruct details of the object smaller than λ,
i.e. smaller than the diffraction or Rayleigh limit

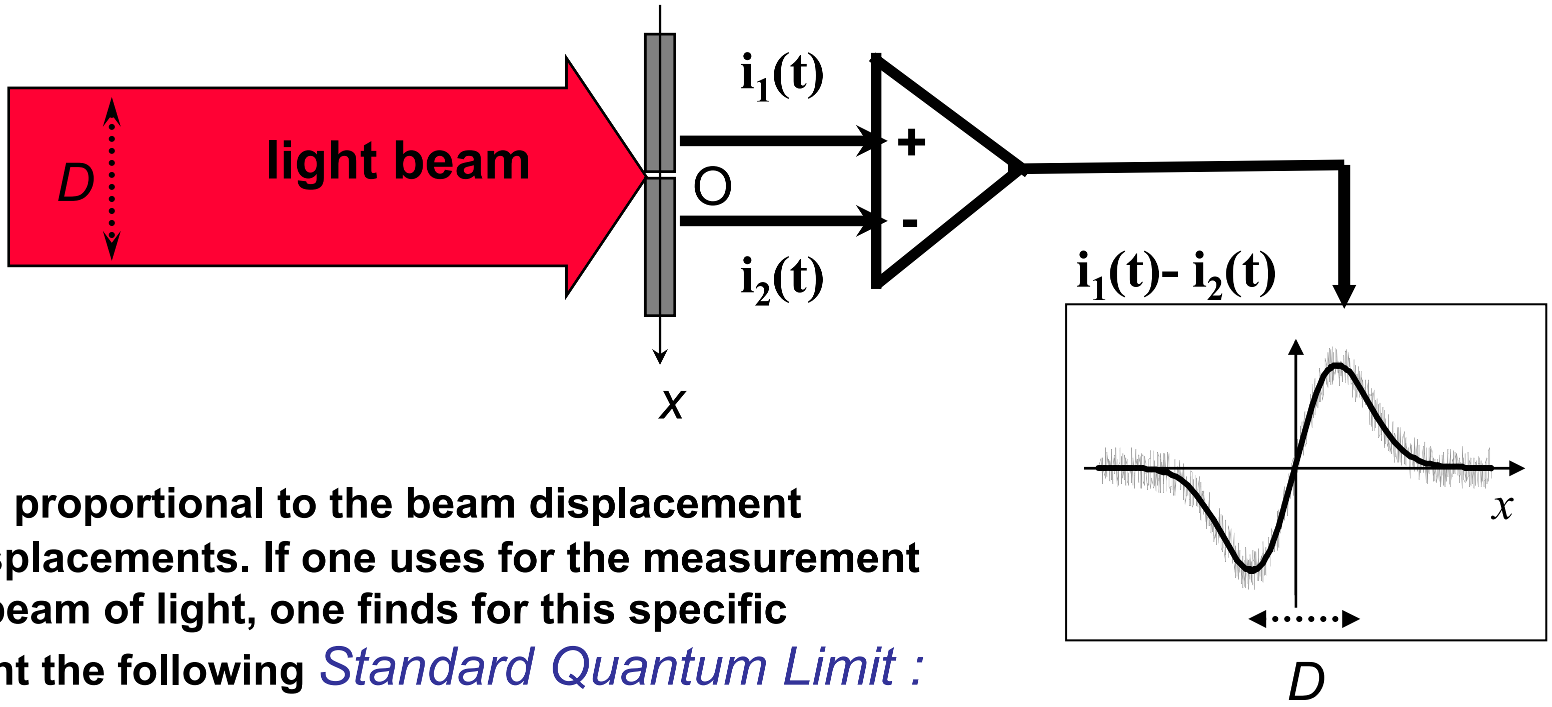
$\Rightarrow$ "superresolution"

in practice, difficult to go beyond a few units

B.

Standard Quantum Limit in information extraction from images

Measurement of very small displacements in the transverse plane



$i_1(t)$- $i_2(t)$ **is proportional to the beam displacement for small displacements. If one uses for the measurement a coherent beam of light, one finds for this specific measurement the following** *Standard Quantum Limit :*

$$\delta x_{\min} = \frac{D}{\sqrt{N_{photons}}}$$

$N_{photons}$: number of photons measured in total beam

it can be much smaller than the wavelength

but $N_{photons}$ **cannot be increased without limit**

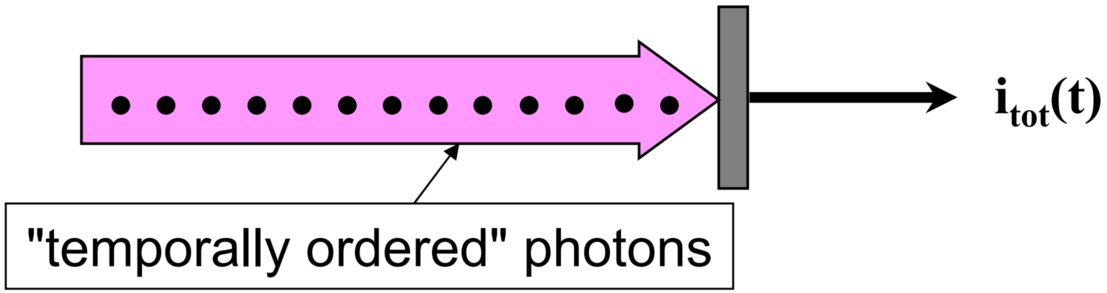
-II-

Beyond the Standard quantum limit

(using "non-classical" beams)

There exist nonclassical states of light
which allow us to go beyond the standard quantum limit
for measurements performed on the total beam

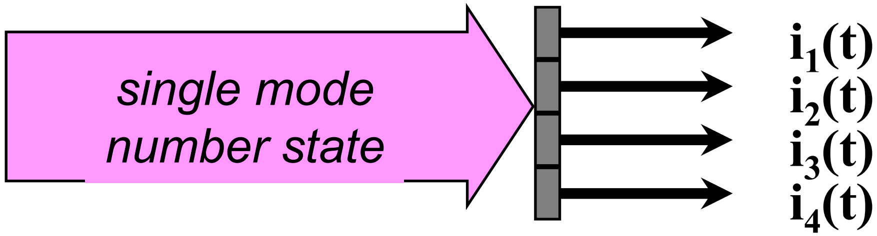
•*number states, sub-Poissonian states for intensity measurements,*



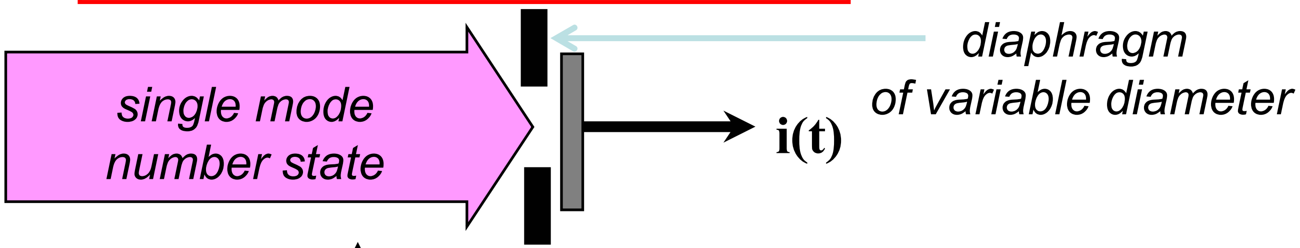
• *squeezed states for quadrature measurements*

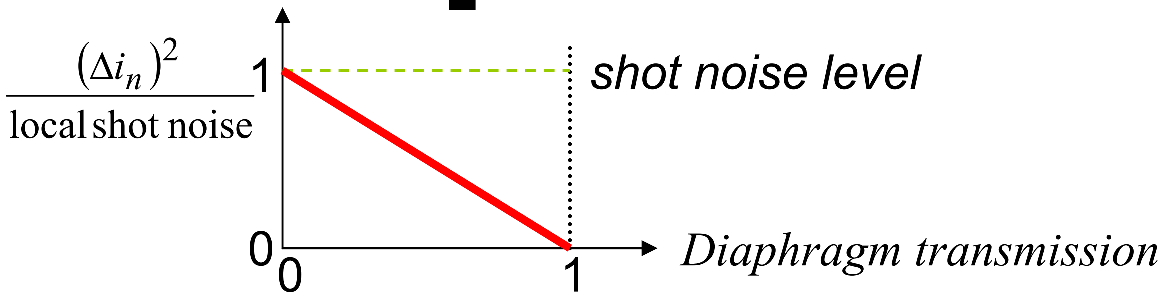
They are single transverse mode non-classical light

Can one use single mode non-classical light to improve imaging ?



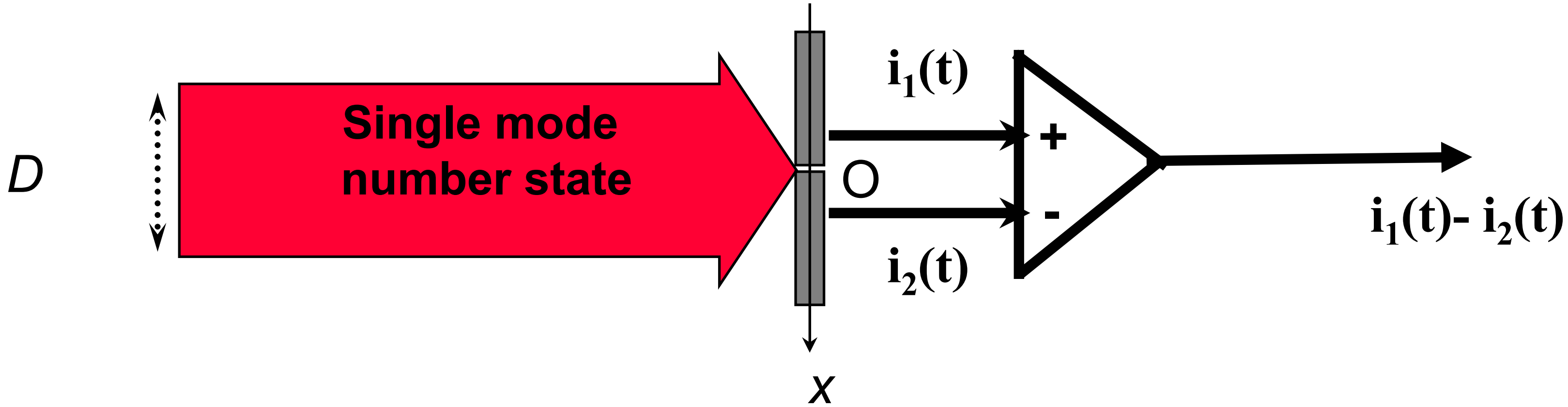
$$(\Delta i_n)^2 = \text{local shot noise} \times \left[1 - \frac{\langle i_n \rangle}{\langle i_{total} \rangle}\right]$$





even the best single mode non-classical state is not helpful
when one wants to measure small details in the transverse plane

Use of a single transverse mode non-classical state to measure a small displacement in the transverse plane



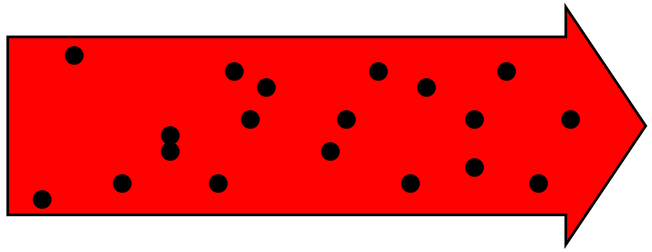
With a single mode number state :

$$\delta x_{\min} = \frac{D}{\sqrt{N_{photons}}}$$

One does not beat the Standard Quantum Limit !

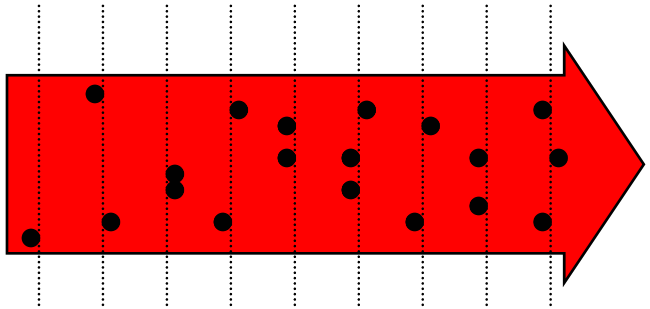
A coherent state beam is «composed» of photons

randomly distributed in space and time



a single mode number state is «composed» of photons

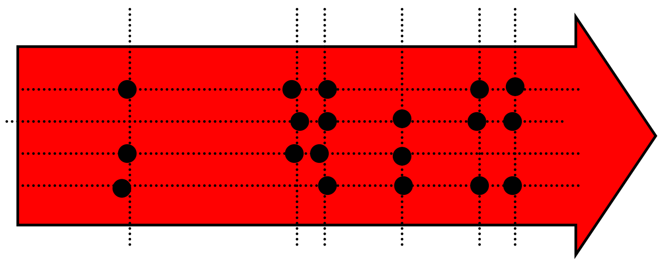
ordered in time but randomly distributed in space



and therefore it cannot improve measurements in which the noise is related to the spatial distribution of photons

For measurements in images, one needs a light beam

with photons **ordered in space**



⇒

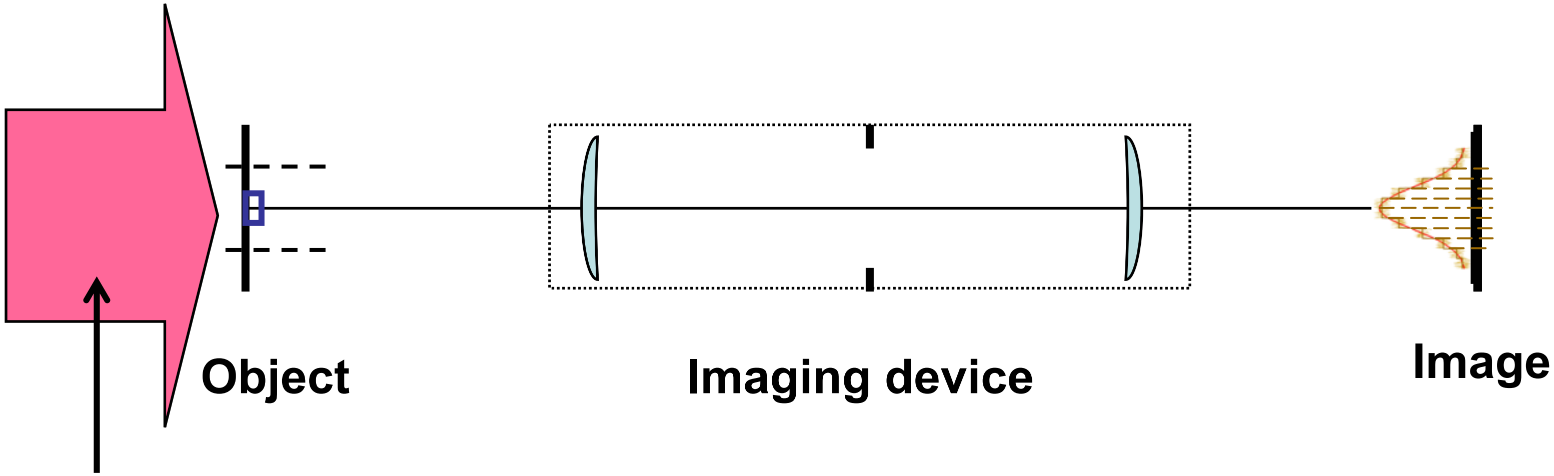
A multimode nonclassical beam
is needed to reduce noise in images

A.

Beyond the Standard Quantum Limit in object reconstruction

M. Kolobov, C. Fabre Phys. Rev. Letters 85, 3789 (2000)

Quantum noise must be reduced in all the eigenmodes f_n of the system



multimode squeezed light improves object reconstruction by pushing the cut-off of spatial frequencies to higher values

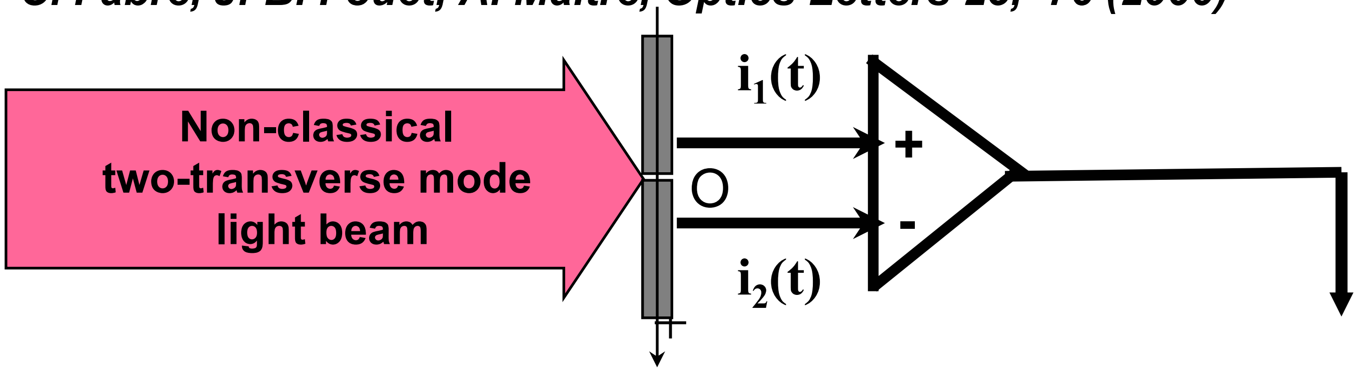
The multimode squeezed light must be emitted by the object, and also around the object, and in all the eigenmodes of the imaging device

B.

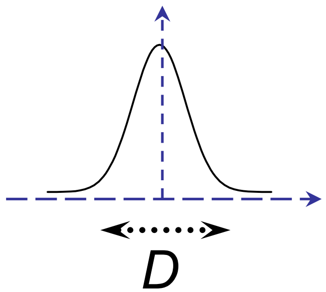
Beyond the Standard Quantum Limit in beam positioning

Beyond the Standard Quantum Limit in 1 D positioning

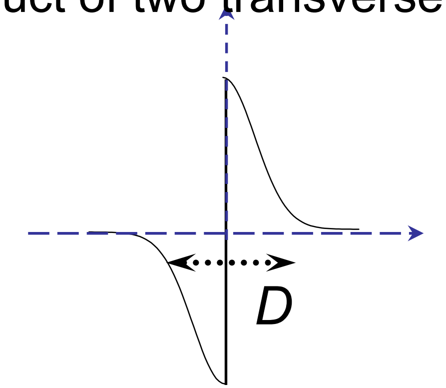
C. Fabre, J.-B. Fouet, A. Maître, Optics Letters 25, 76 (2000)



One finds the beam likely to reduce the noise
in such a measurement is a tensor product of two transverse modes :



and

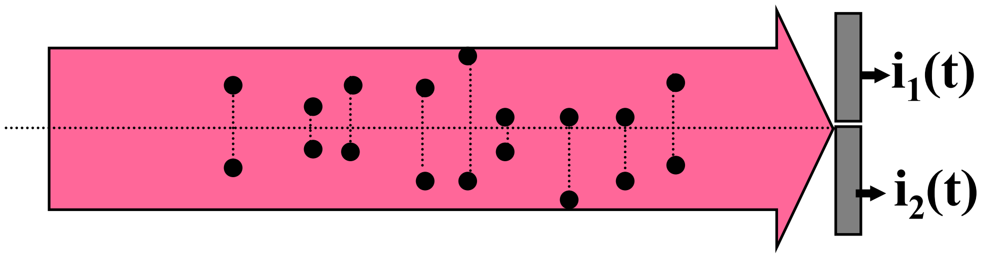


perfectly squeezed vacuum in TEM_{00} mode $\otimes$ intense coherent state in "flipped" TEM_{00} mode

$$\delta x_{\min} \ll \delta x_{SQL}$$

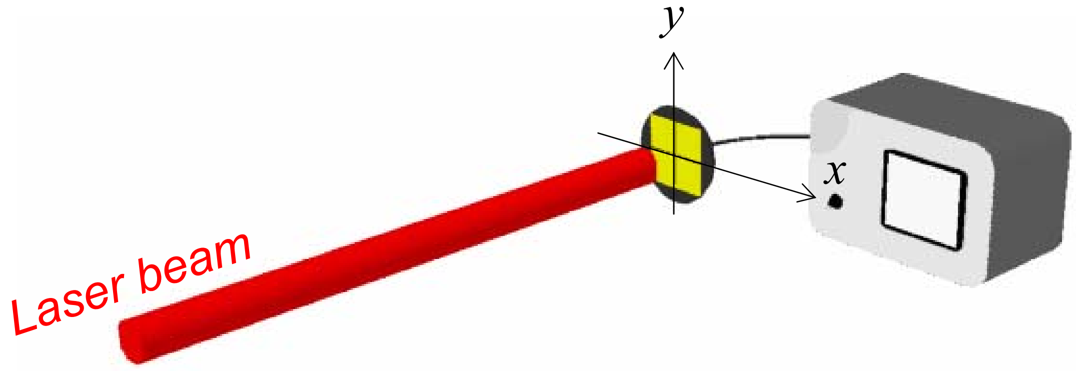
- in a single mode squeezed beam, the two halves of the beam are more noisy than the total beam : the fluctuations on the two halves are anticorrelated

- the mixing with a "flipped "mode transforms the anti-correlation into a correlation



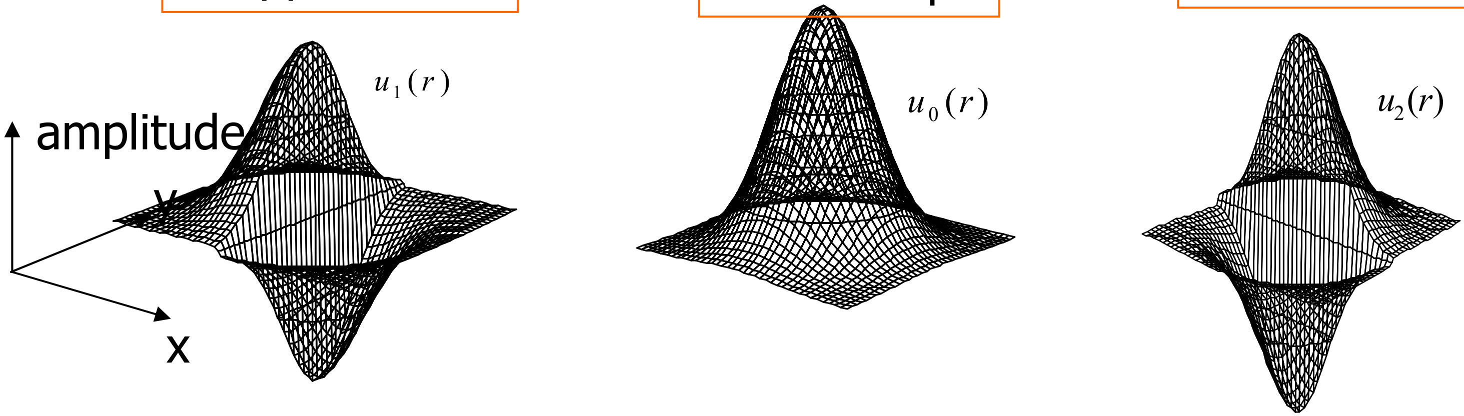
⇨ photons are spatially ordered two by two, in the two halves of the beam

Beyond the Standard Quantum Limit in 2 D positioning
"the quantum laser pointer"

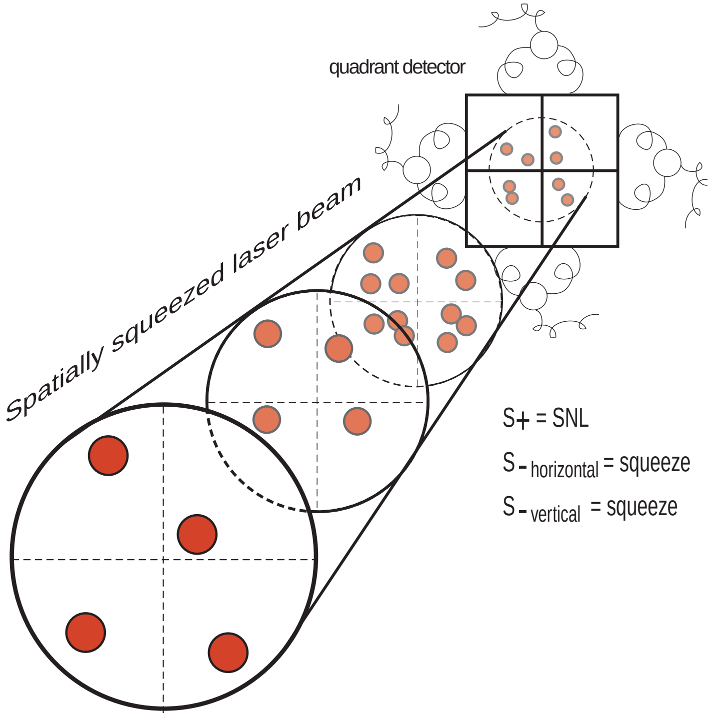


The beam likely to reduce the noise in a 2D positioning measurement is a tensor product of three transverse modes :

x flipped mode | Beam shape | y flipped mode



squeezed vacuum $\otimes$ *coherent state* $\otimes$ *squeezed vacuum*



In such a non-classical state, the photons are "ordered 4 by 4", in four quarters of the beam

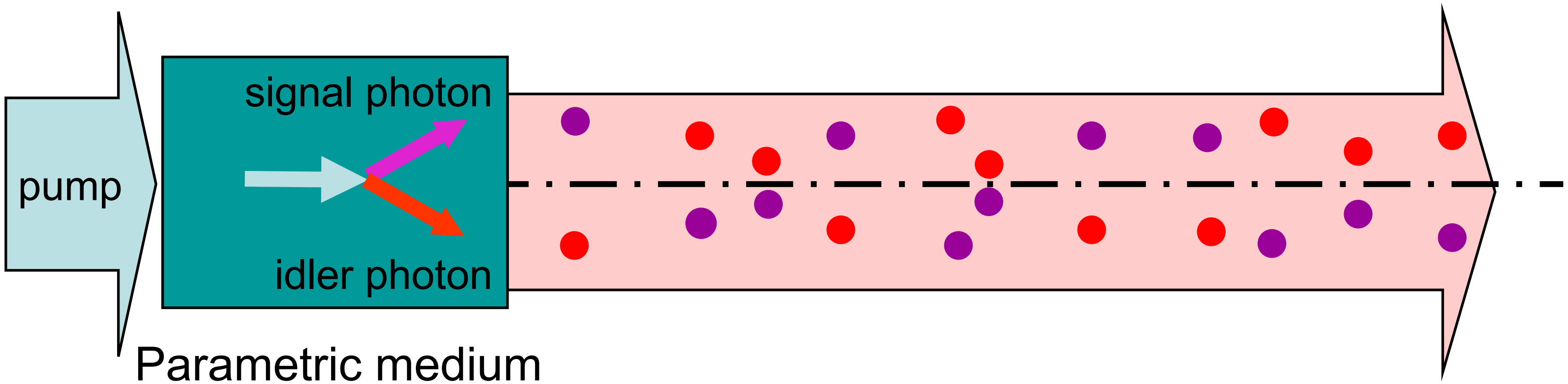
-III-

How to generate

multimode non-classical beams ?

A.

Direct generation using nonlinear optics parametric down conversion is a good candidate to generate spatial non-classical effects



If the pump is a plane wave, the signal and idler photons are emitted at the same time and in perfectly correlated directions

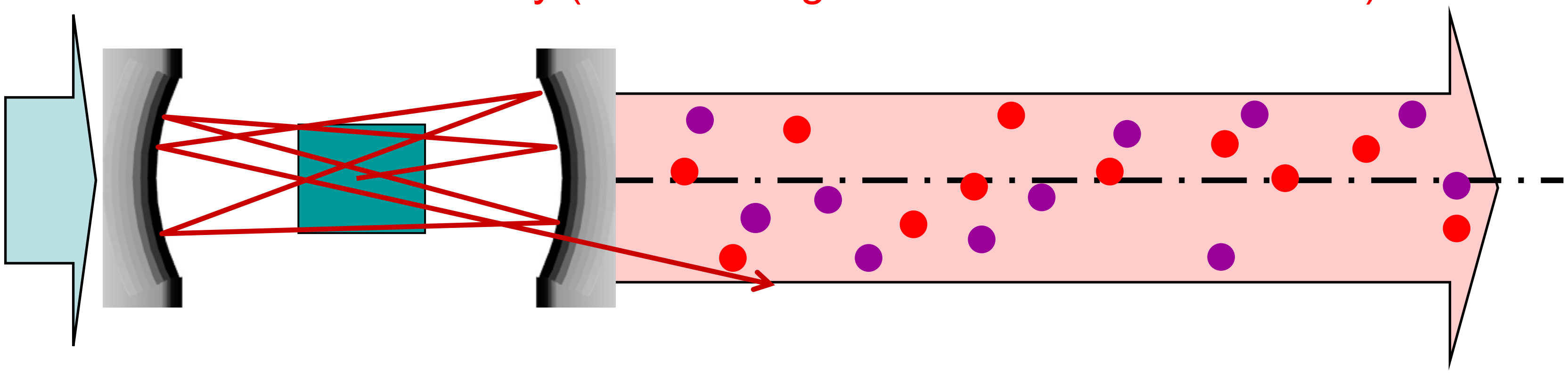
⇒ temporal and spatial quantum correlations

1) regime of high parametric gain using intense pulsed pump laser

in the frequency degenerate case
multimode squeezed light is produced

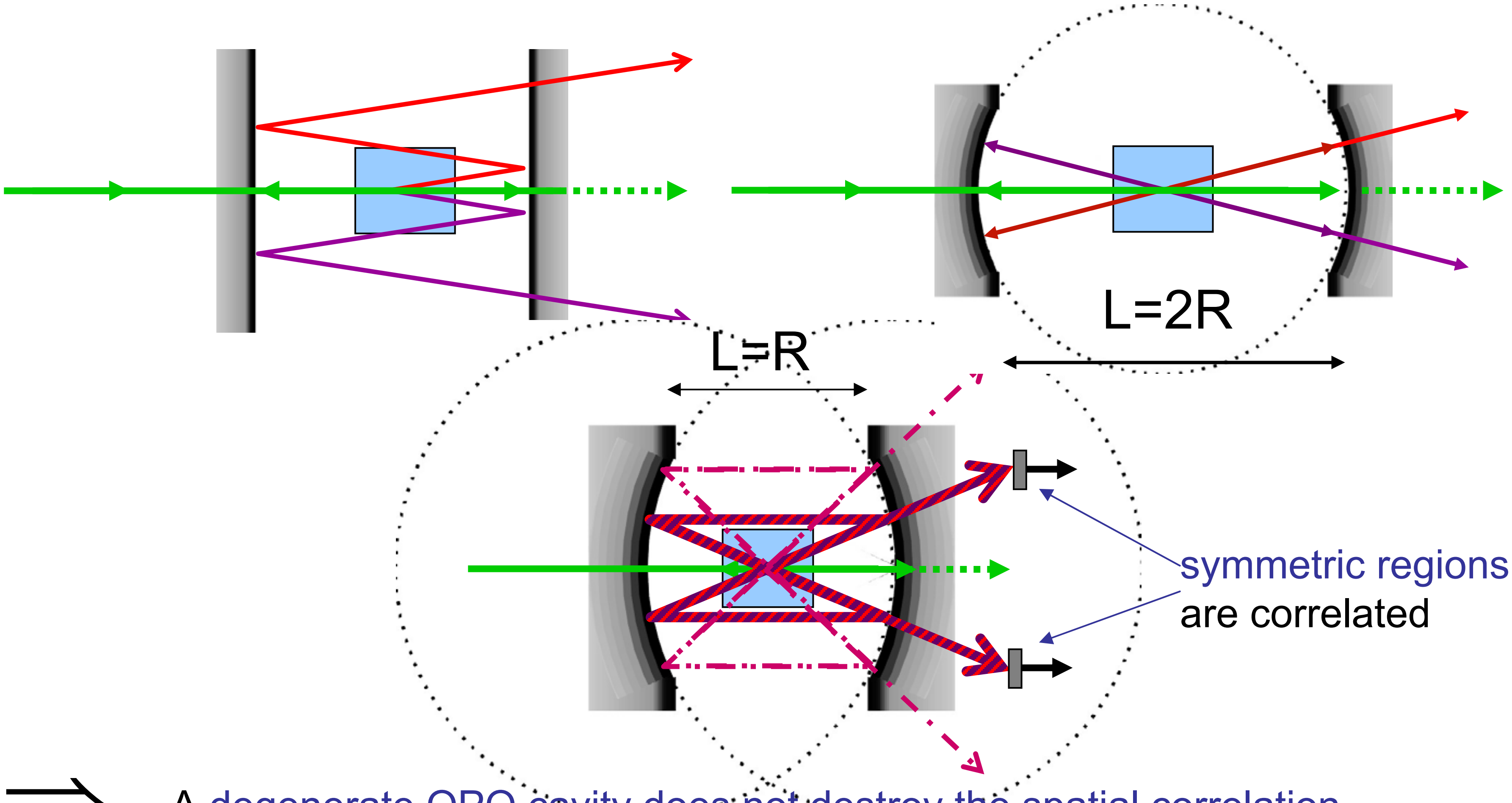
2) regime of parametric oscillation in an optical cavity

with a usual cavity (without degenerate transverse modes)



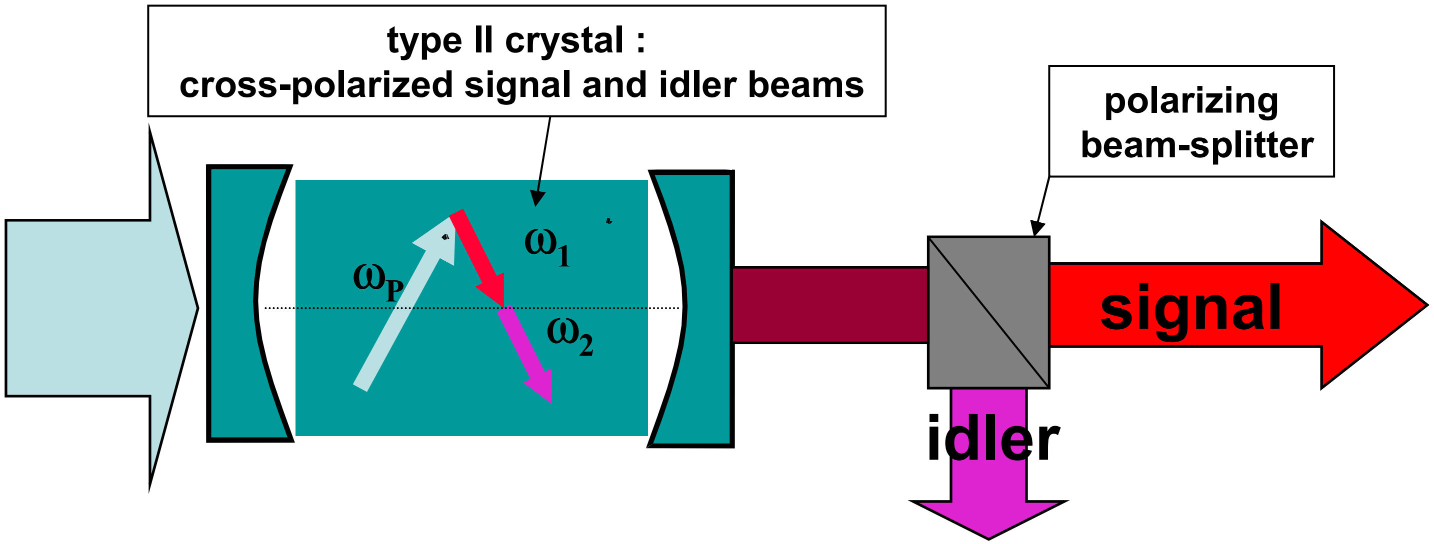
Because of multiple reflexions on the curved mirrors,
a regular non-degenerate OPO cavity
destroys the spatial correlation created in parametric down-conversion

with a cavity having degenerate transverse modes (planar, concentric, confocal)



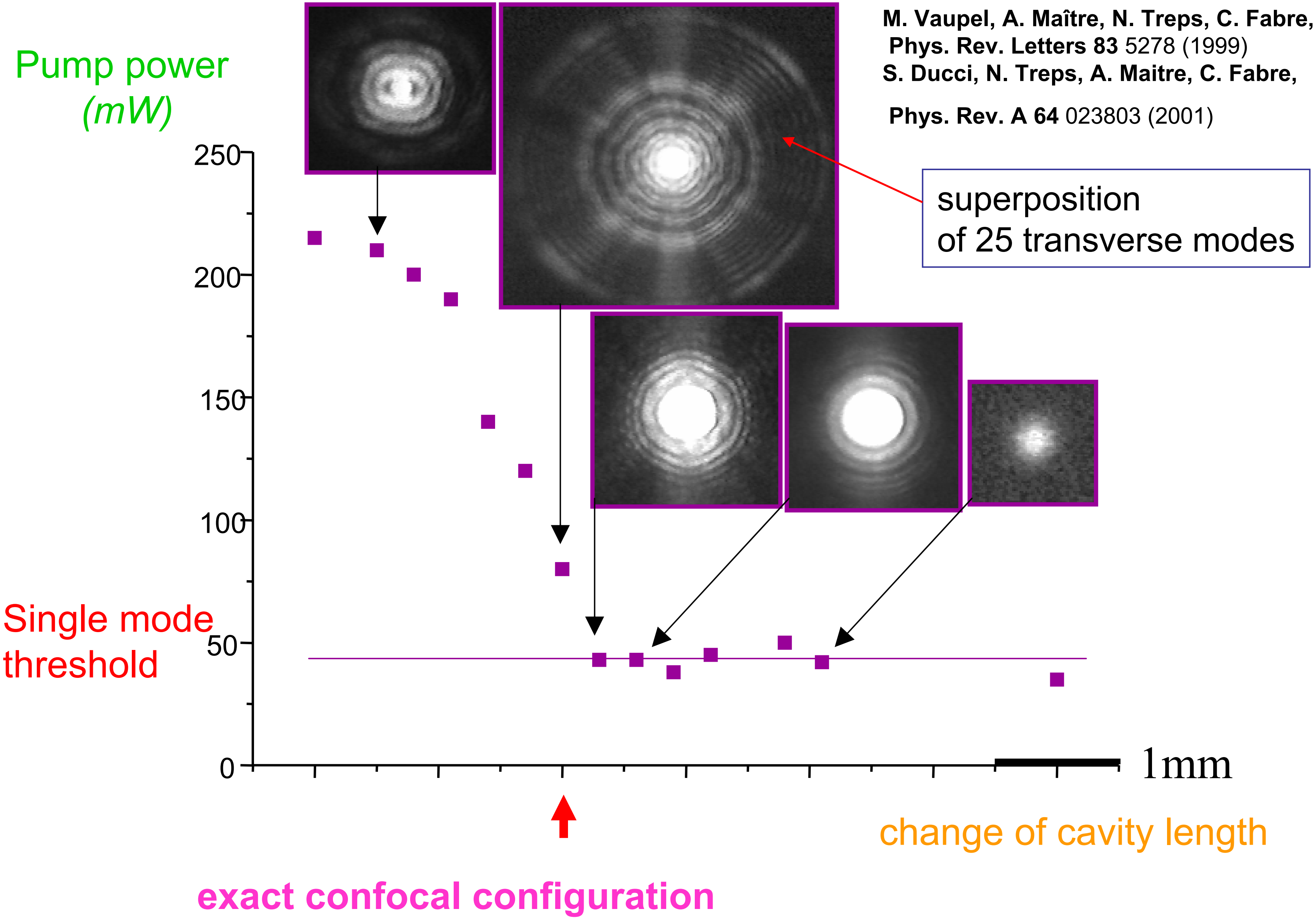
⇒ A degenerate OPO cavity does not destroy the spatial correlation created in parametric down-conversion

Experiment in Paris :
evidence for direct generation of multi-mode nonclassical state of light in a type II confocal OPO

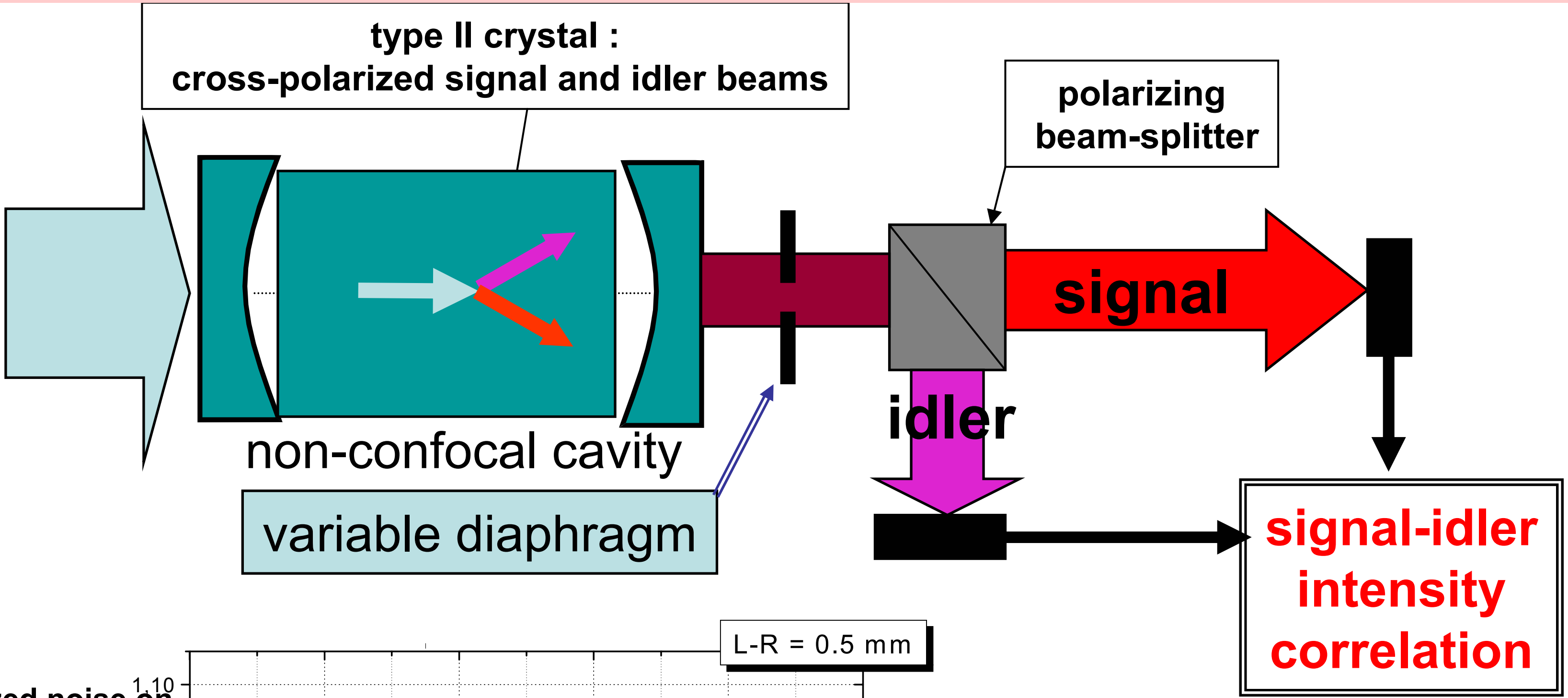


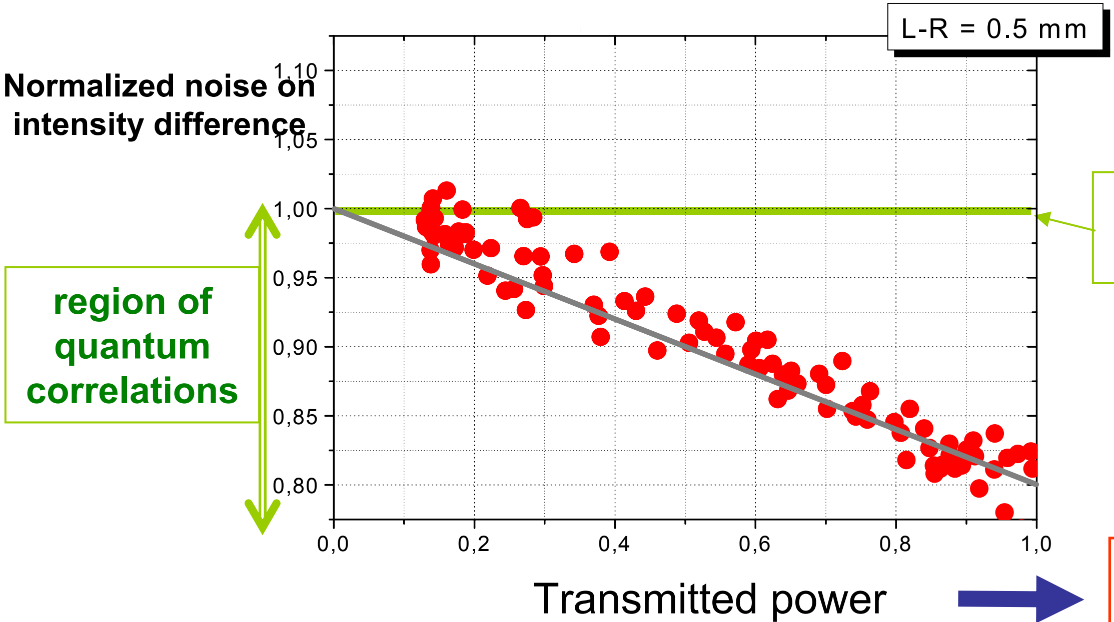
OPO operation in a confocal cavity : generation of optical patterns

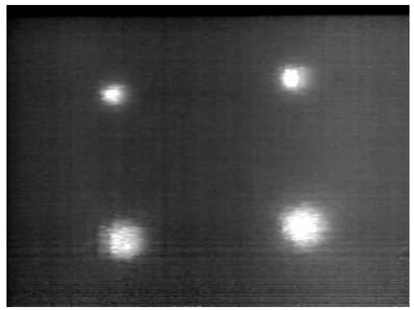
M. Vaupel, A. Maître, N. Treps, C. Fabre, Phys. Rev. Letters 83 5278 (1999)
S. Ducci, N. Treps, A. Maitre, C. Fabre, Phys. Rev. A 64 023803 (2001)



study of the spatial distribution of intensity correlations in a non confocal OPO



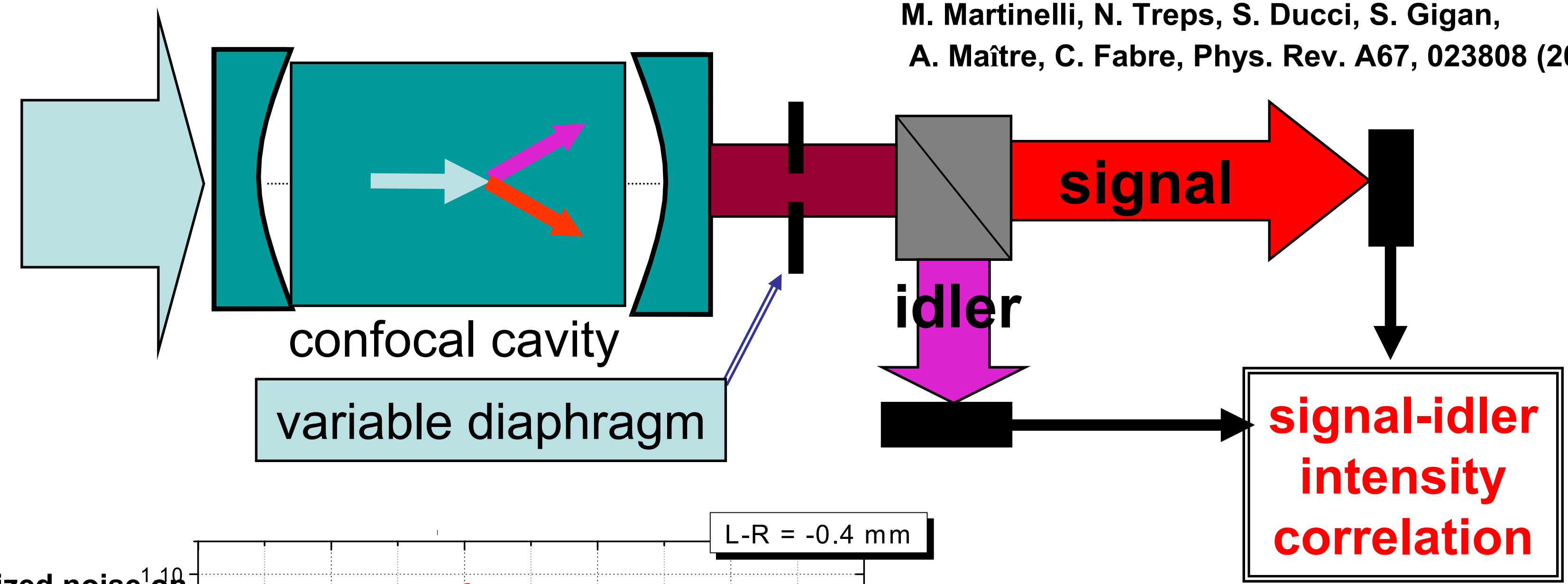


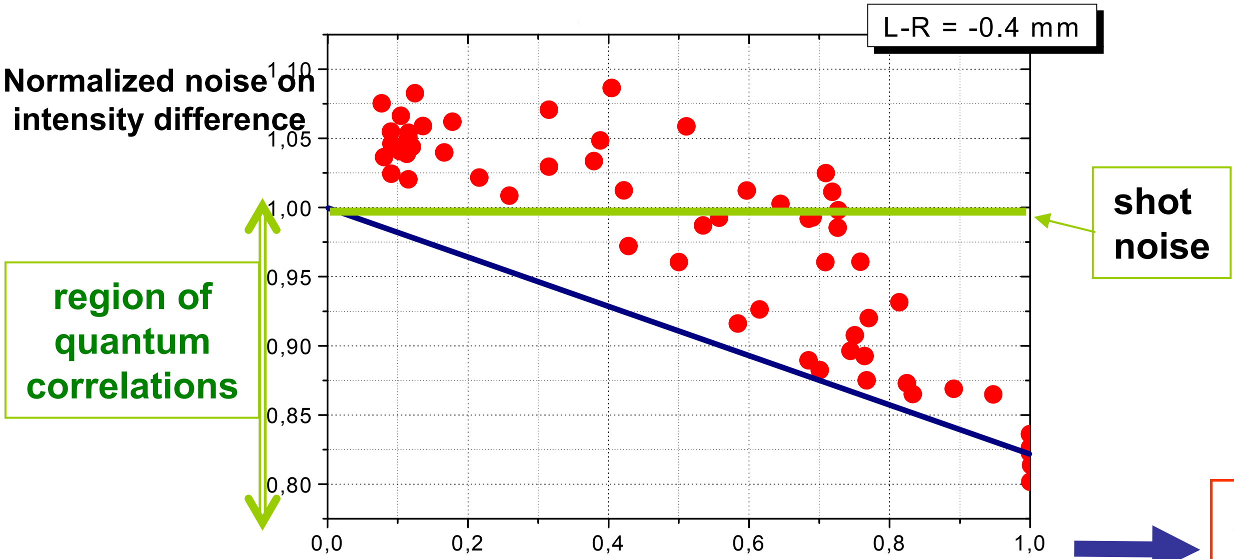


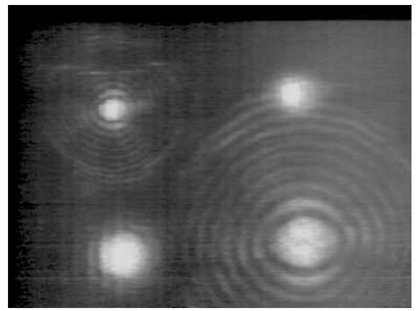
signature of a single mode non classical state of light

study of the spatial distribution of intensity correlations in a confocal OPO

M. Martinelli, N. Treps, S. Ducci, S. Gigan, A. Maître, C. Fabre, Phys. Rev. A67, 023808 (2003).







the outer part of the beams
are quantum correlated, not the central part

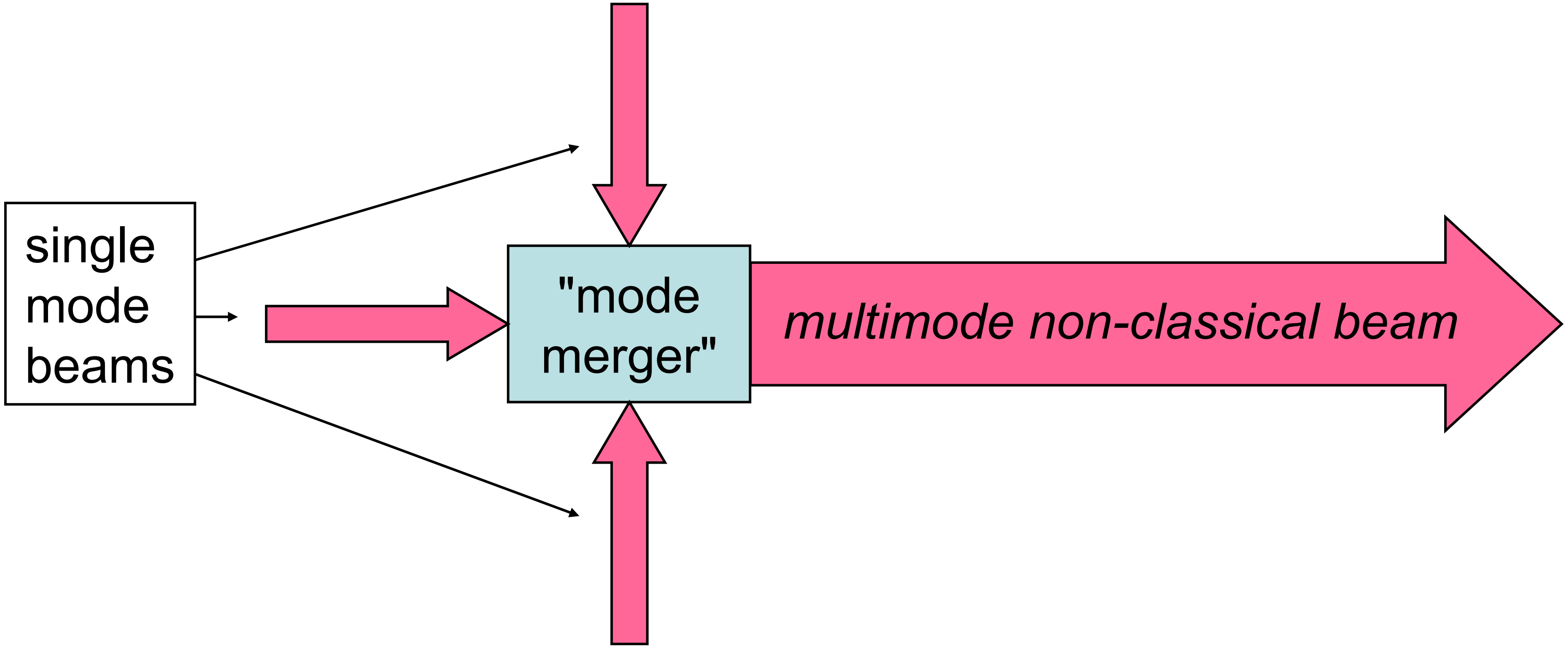
signature of a ***multimode*** *non classical state of light*

B.

Synthesis of multimode non-classical beam

from single mode non classical beams

collaboration Paris- LKB Australian National University (Canberra)



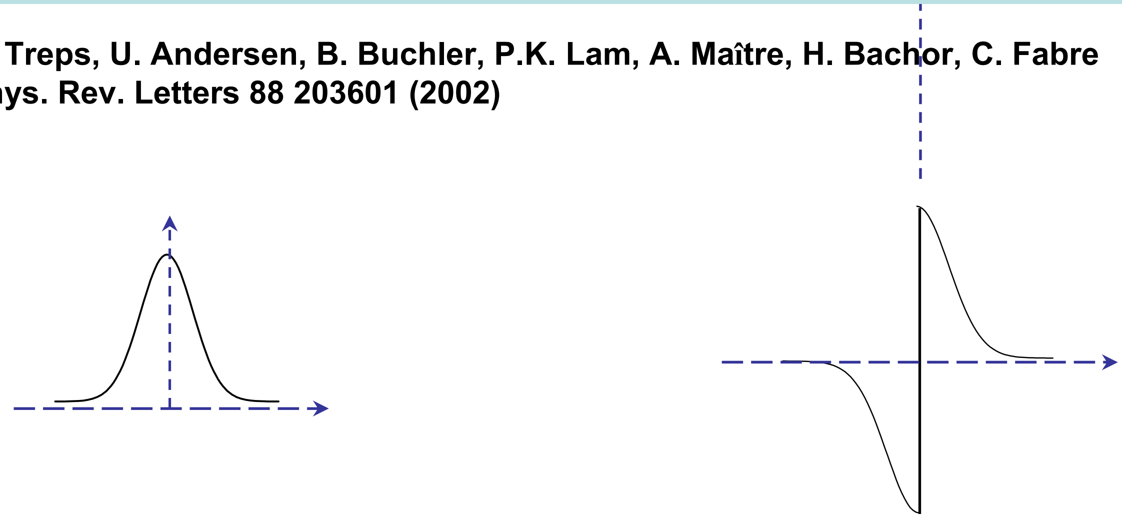
a mode merger can be :

• a beamsplitter : simple, but introduces losses and therefore excess noise

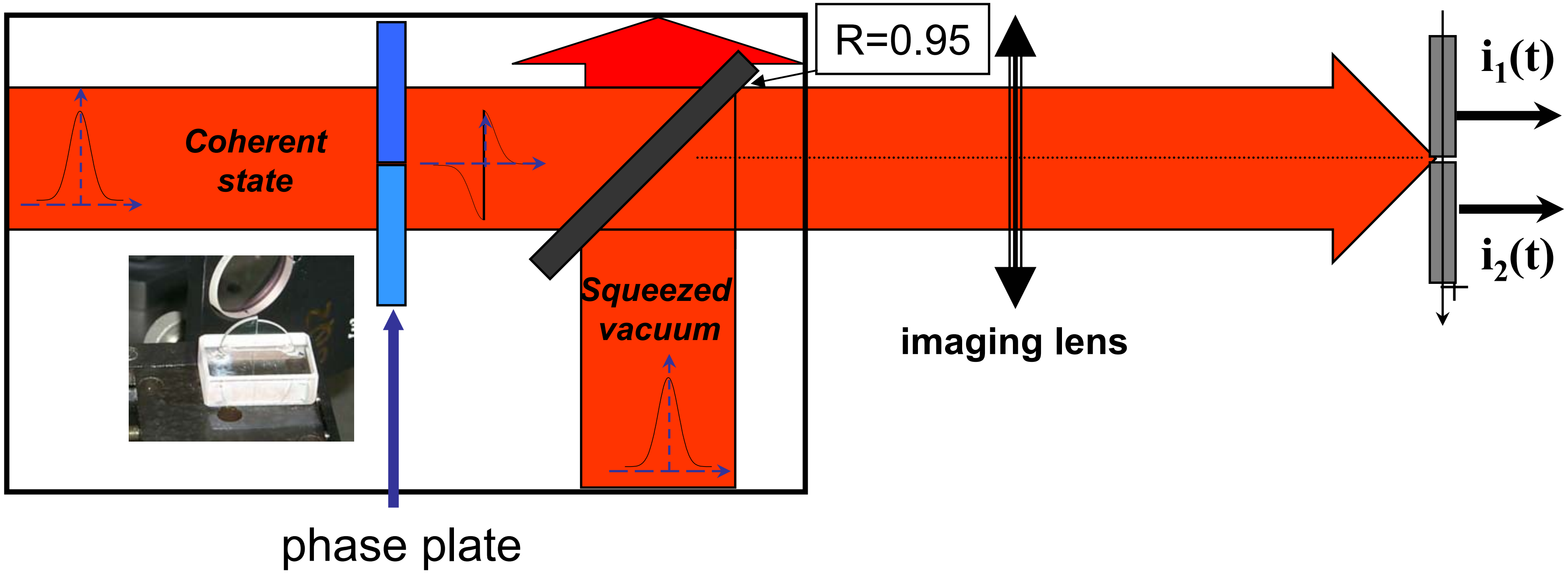
• an optical cavity transmitting a transverse mode and reflecting the others

Experimental production of a non-classical two-transverse mode light beam

N. Treps, U. Andersen, B. Buchler, P.K. Lam, A. Maître, H. Bachor, C. Fabre
Phys. Rev. Letters 88 203601 (2002)

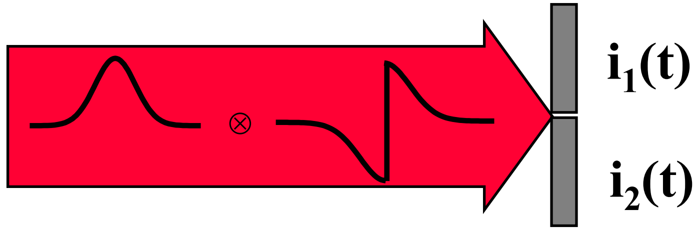


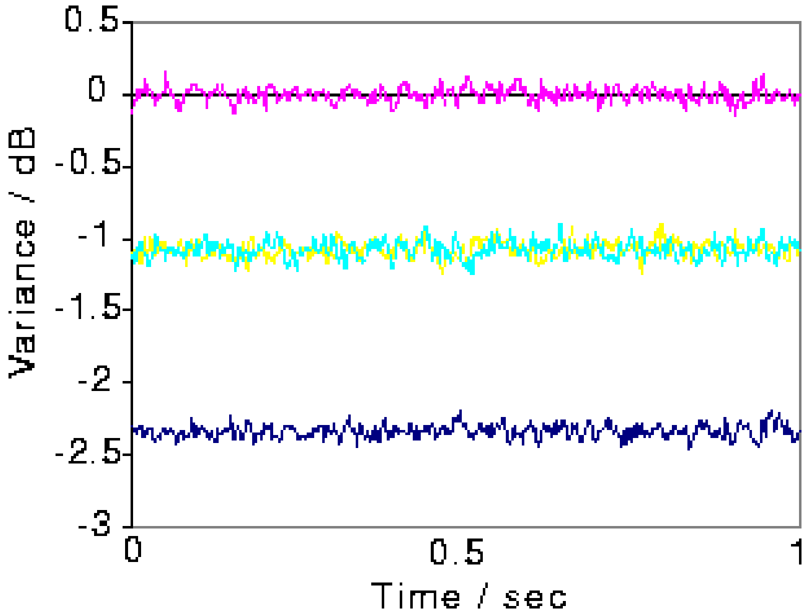
perfectly squeezed vacuum $\otimes$ Intense coherent state



Evidence of non-classical spatial effect

Analysis frequency : 4.5 MHz



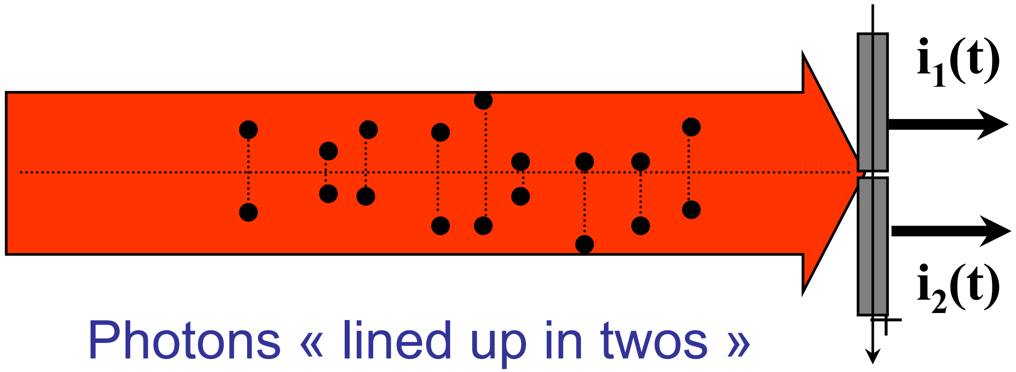


total signal $i_1(t) + i_2(t)$: shot noise level

noise on $i_1(t)$ or $i_2(t)$ **-1.08 dB**

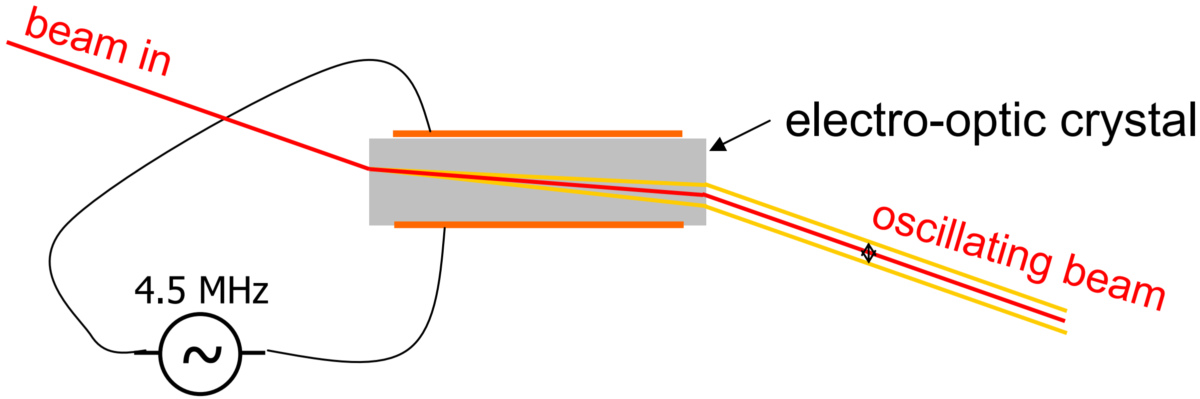
noise on $i_1(t) - i_2(t)$ **-2.34 dB**

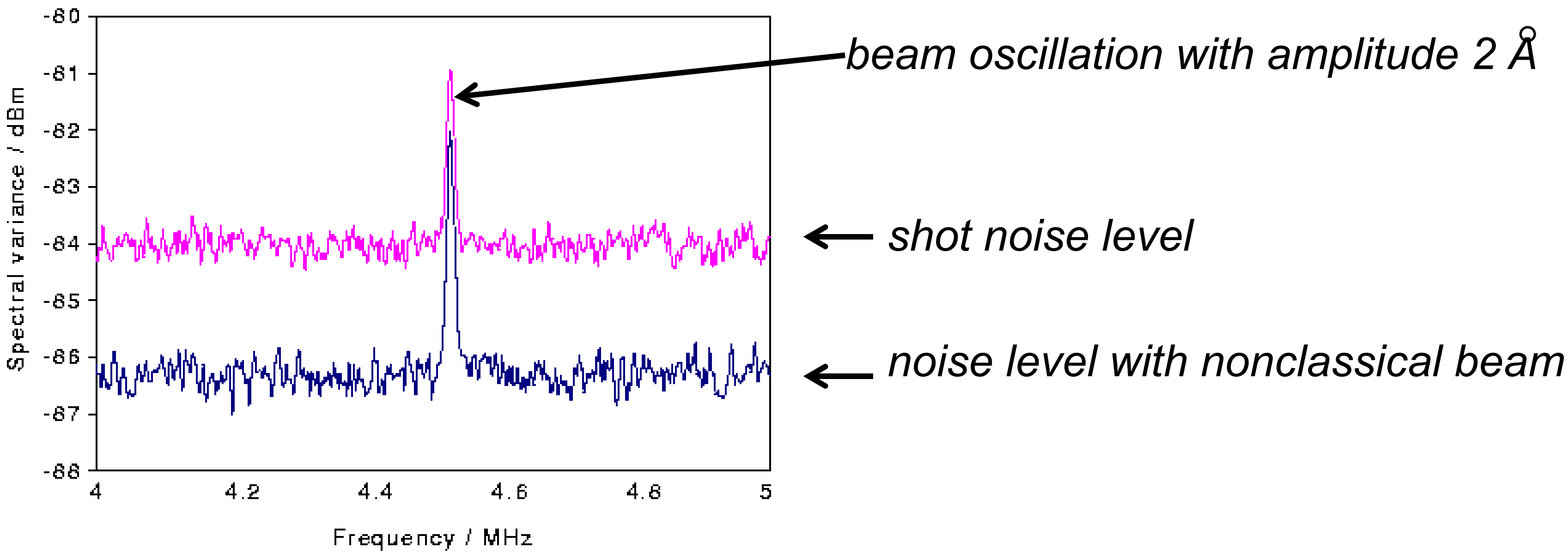
First light beam produced with nonclassical spatial effect,



Photons « lined up in twos »

1D small displacement measurement



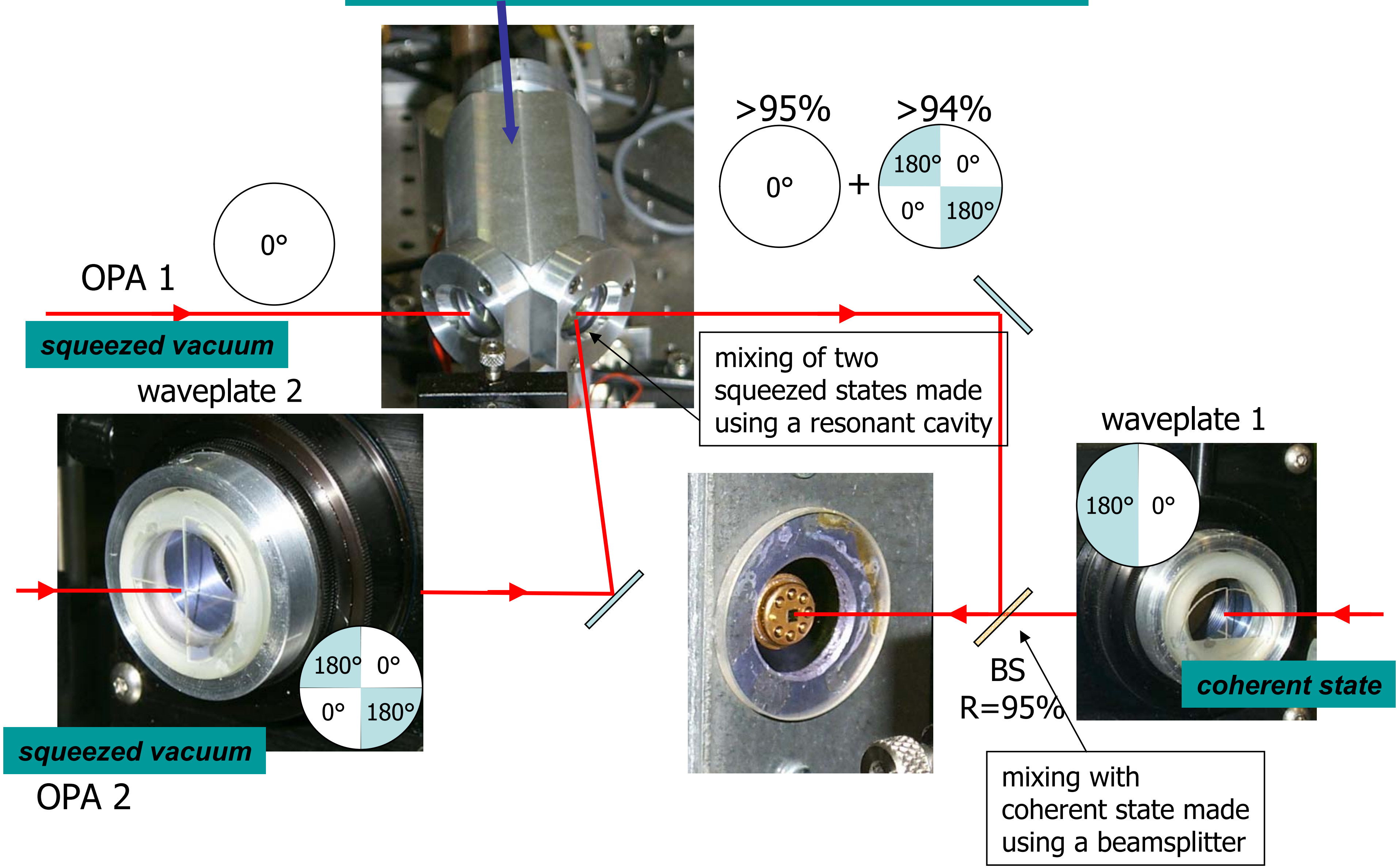


RBW=10 kHz

signal/noise enhancement factor : 1.7

Production of non-classical three-transverse mode light beam

High finesse Fabry-Perot ring cavity :
transmits TEM_{00} mode and reflects all the others

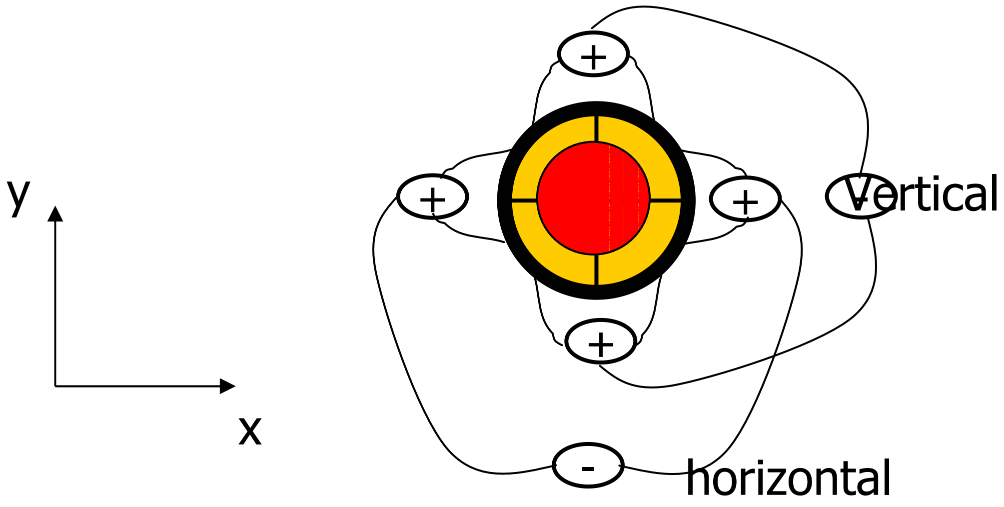


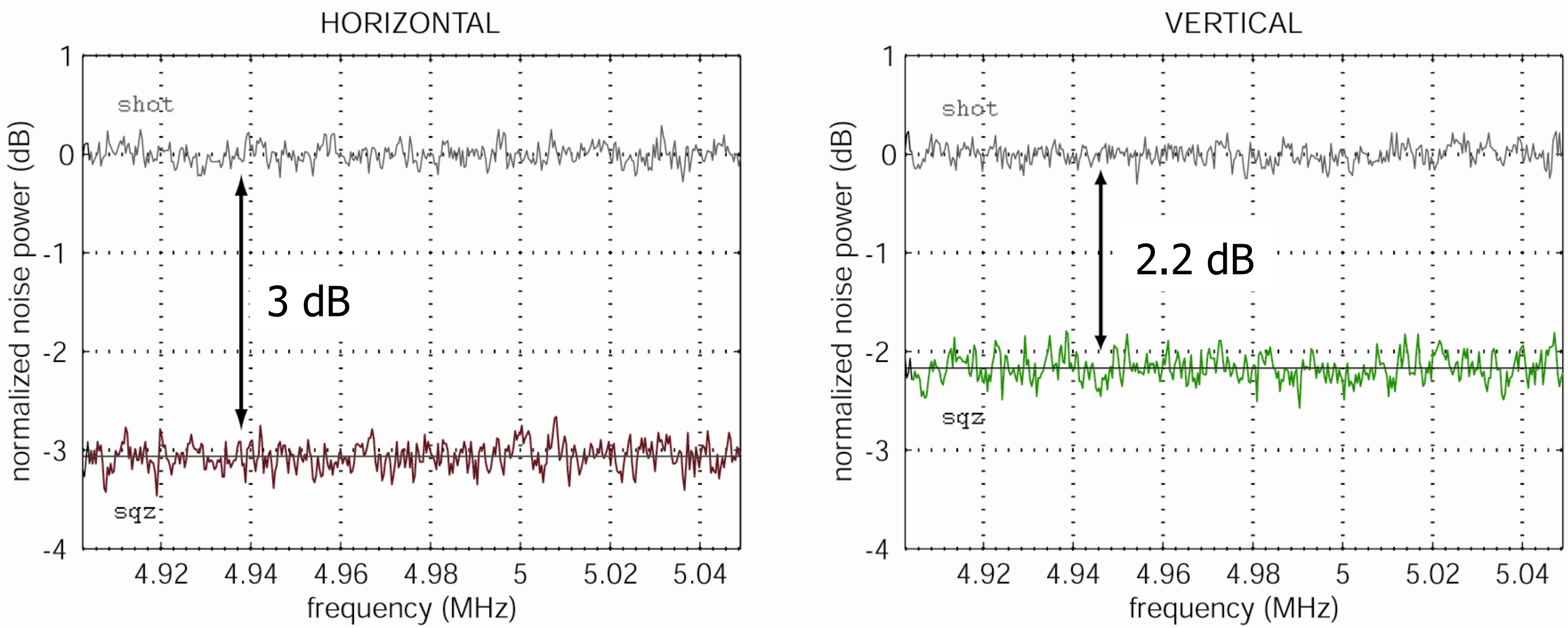
First beam ever produced with

- multimode transverse squeezing

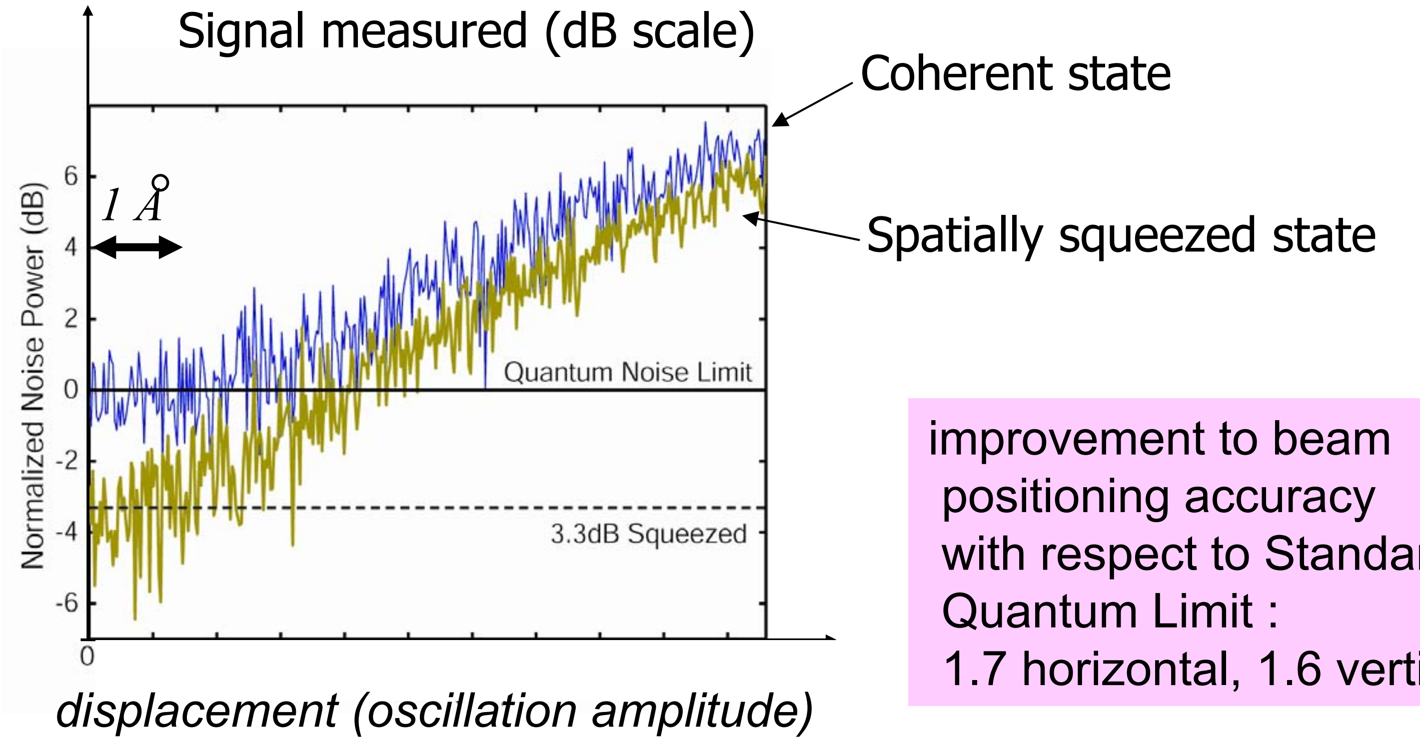
- 4 quantum correlated zones

- two "quantum channels" encoded in transverse plane





Used to measure at the same time displacements in two dimensions below the shot noise limit



improvement to beam positioning accuracy with respect to Standard Quantum Limit :
1.7 horizontal, 1.6 vertical

N. Treps, N. Grosse, W. Bowen, C. Fabre, H. Bachor, P.K. Lam
"The quantum laser pointer", Science 301*, 940 (****2003****)*

Conclusion

⇒ **Quantum noise, more than diffraction, gives the ultimate limit to the sensitivity of information extraction in an image**

⇒ **"Ordinary", single mode squeezed light is not useful to enhance the sensitivity of this kind of measurement**

⇒ **Experimental implementations :**
- using two- or three- mode nonclassical state
- using multimode OPOs